English Light Orchestra
a module on patterns and expressions

Teacher Guide
CORNERSTONE MATHS
About the cover
Once upon a time, a design collective came upon the idea of creating art installations made of lights and developed the English Light Orchestra campaign.

The stories in this work are fictional. All characters and events appearing in this work are fictitious. Any resemblance to real persons, living or dead, is purely coincidental.

English Light Orchestra - A unit on patterns and expressions
Developed jointly by SRI International, Center for Technology in Learning (United States) & London Knowledge Lab, Institute of Education, University of London (UK)

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Preface to the Teacher's Edition

Unit Overview

This unit supports pupils to understand and express algebraic generalisations arising from how they visualise the structure of a figural pattern. Pupils work in a dynamic environment to visualise and generate figural patterns and write algebraic expressions for those patterns. The activities encourage pupils to identify structures in the patterns to make sense of the letters and numbers in an algebraic expression. One of the main goals of the unit is to help pupils develop a solid understanding of variables as representing quantities that vary instead of simply unknowns to be discovered.

Working with patterns to develop concepts in algebra is common, but this unit uses patterns a bit differently. Rather than present static images of “growing” figures in order from first to nth, we present figures dynamically and in random order, so that pupils cannot simply apply additive reasoning to develop an expression for the number of elements in the nth figure without understanding the meaning of the variable in that expression.

In developing expressions to represent the total number of (in our case) lights in any figure, pupils first work with numbers and operations, and then choose which numbers to “unlock”, based on what the number represents. Depending on how they see and analyse a pattern, different pupils can come up with different expressions for the same pattern. The unit provides opportunity for pupils to grapple with what makes these expressions equivalent, at an informal level. Tables and graphs are used as supporting representations by providing evidence that two expressions are equivalent (or not).

Mathematics Goals

Pupils will understand that algebraic expressions can express the structure of repeating figures (figural patterns) and be used to determine the number of elements in the nth term of those patterns.

Pupils will understand the role of variable in an algebraic expression.

Pupils will be able to:

• Recognise that the same pattern can lead to multiple equivalent expressions.

• Write multiple algebraic expressions for a linear figural pattern that are equivalent to $an + c$, where $c$ may sometimes be zero.

• Generate multiple patterns for the same linear algebraic expression.

• Explain why two expressions are equivalent in terms of the structure of a pattern.

• Use tables, graphs, and expressions to help confirm equivalency or establish non-equivalency of patterns.
**National Curriculum Addressed**

**National Curriculum September 2013**
One of the aims of the National Curriculum for mathematics is for all pupils to reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.

**Key Processes**
At Key Stage 3, through the mathematics content pupils should be taught to:

**Develop fluency**
- use algebra to generalise the structure of arithmetic, including to formulate mathematical relationships
- substitute values in expressions, rearrange and simplify expressions, and solve equations
- move freely between different numerical, algebraic, graphical and diagrammatic representations
- develop algebraic and graphical fluency
- use language precisely to analyse algebraic expressions

**Reason mathematically**
- extend their understanding of the number system, make connections between number relationships, and their algebraic and graphical representations
- identify variables and express relations between them algebraically and graphically

**Solve problems**
- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes
- develop their use of formal mathematical knowledge to solve and devise problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations

**Subject content**

**Algebra:**
Pupils should be taught to:
- read and interpret algebraic notation;
- substitute numerical values into formulae and expressions;
- understand and use the concepts and vocabulary of terms and expressions;
- simplify and manipulate algebraic expressions to maintain equivalence by collecting like terms;
- model situations or procedures by translating them into algebraic expressions and by using graphs;
- recognise, sketch and produce graphs of linear functions of one variable with appropriate scaling, using equations in x and y and the cartesian plane;
- interpret mathematical relationships both algebraically and graphically.
Implementation Suggestions

Materials
- Pupil book – either the Workbook where pupils write their answers in their booklet
  OR the Book, which can be reused.
- Pencil, coloured pencils and rough paper.
- Tablets / computers for pupils.
- Whole class display: Interactive whiteboard, computer with projector, or document camera

Classroom Organisation
Pupils work in groups of two/three, around one device (computer, tablet). For some of the activities, you can regroup pupils, jigsaw style, with members of two other groups.

Many activities begin with a whole class discussion that launches pupils into work in groups and then pupils come together again for plenary discussion. When recommending whole-class, group work, independent work, or collaborations, these are our assumptions:

1. Whole class discussion, teacher leads but pupil actively participate.
   - Whole class display/use of the software is used to support discussions.
   - Everyone can hear everything that everyone says.
   - Each pupil should complete work in his/her own Workbook (or exercise book/paper).

2. Group work, teacher circulates while pupils work in groups of two/three (or two if necessary).
   - Pupils are seated so each group member can see the shared computer or tablet screen.
   - Pupils should work together to come up with common solutions.
   - Each pupil should complete work in his/her own Workbook (or exercise book/paper).

3. Homework or independent work.
   - Pupils can work alone or in groups.
   - Focused on practice of content in activity or remembering old material relevant for the next day’s activity.

4. Collaborations (Online group tasks using the software)
   - Pupils work in groups, preferably groups of three.
   - Pupils are seated so each group member can see the shared computer or tablet screen.
   - Pupils work together to build a correct solution and justify their choices.
   - The software provides feedback on the correctness of their solution.
General Teaching Tips

- Allocate additional time for pupils to become familiar with the software during the first investigation and revisit the important ideas (unlocking, linking, building expressions) in the early lessons to ensure that they (and you) can use the software confidently.
- Take time to discuss and establish the mathematical and technological vocabulary - and revisit this.
- Encourage lots of discussion and once pupils have rehearsed their thinking, prompt them to write it down!
- Facilitate the pupils’ productive struggle. There are key situations where pupils should grapple with the mathematics prior to being shown how to solve the problem at hand.
- Encourage explanations of correct and incorrect answers.
- Help pupils to make connections among multiple correct answers.
- Let pupils do more talking than you do. During whole class mode, let them answer questions and challenge the answers others give. You can use your own strategies for engaging pupils in the plenary.
- Balance whole class work with individual and group work. Pupils need both.

Rich Opportunities for Learning

Pupils may first think of a variable as a letter that stands for a single missing number, based on their prior school experience. You can help them transition to understanding variables as symbols that can be replaced by a set of (potentially infinite) numbers. The software is designed precisely to help pupils develop a richer concept of variable—through encouraging pupils to consider a variable in relation to a set of numbers (between 1 and 10), building expression(s) using that variable, and observing how a changing pattern is represented (or not) by the expression(s).

Pupils may develop or come to class with faulty rules for simplifying expressions. For example, they may think that $n + 2n$ is equivalent to $2n$, because “there is no number” in front of the first $n$. Or that $2n+3$ is equivalent to $5n$, because the numbers can be added. Through comparing expressions that correctly represent the same situation (figural pattern), they will learn about equivalent expressions. Through generalizing across equivalent expressions, they will develop a set of rules for simplifying expressions.

Differentiation

There are a number of strategies you can use to meet the needs of learners with different levels of achievement. Most of these strategies are inspired by the work of Carol Ann Tomlinson (2003), but they rely on other resources as well.

Group work. You can arrange groups in several ways, which have different affordances for pupil learning. In the beginning of the unit, you may want heterogeneous (mixed ability) groups, as pupils are beginning to grapple with new concepts. The challenge for all comes from the materials themselves, and stronger pupils can help weaker pupils (and it may be a surprise who is which in Cornerstone Maths units). Later in the unit, when focusing on a specific concept, you may want homogeneous (similar ability) grouping so that you can help specific groups of pupils who are likely to have difficulties with the same concept.
Flexible teacher. You can use different modalities to reach a broad group of pupils. The unit requires the use of multiple representations. You may emphasize one representation for pupils who are particularly capable with it, or for those pupils who would benefit from seeing things in a different way. You could, for example, provide a handout with blank tables for a pupil who is struggling to see how different expressions might relate to the same figural pattern.

Pupils with high achievement. It is best to try to keep the class together and use the software tabs to extend pupils ideas, rather than encouraging pupils to move to the next Investigation. However, pupils who work at a much faster pace may need extension activities. Although the patterns in the unit are all about numbers of coloured lights, other kinds of visual patterns may elicit different reasoning. For example, you could use the idea of perimeter, as in seating plans for \( n \) tables, to develop other patterns. There are numerous examples of different kinds of patterns available in released test items.

Pupils who struggle. You can use grouping strategies during group work to support different teaching activities. If you know that you will be short on time to visit every group, you can pair a stronger pupil with two weaker pupils. On the other hand, if you know you want to focus on a particular concept two pupils are having trouble with, then you can group those pupils together. In some cases, pupils who struggle can benefit from strategies designed for stronger pupils. Weaker pupils could learn a lot by making a poster about the different expressions that could represent one pattern, for example.

Collaborations

Collaborative learning has a long track record of success in educational research. This Cornerstone Maths unit integrates collaborative learning activities (Collaborations) within Investigations 4 and 5. The collaborations are designed so that an individual must rely on his or her group members to move forward (positive interdependence) and so that a group cannot succeed without each pupil’s individual contribution (individual accountability). The collaborations also provide an opportunity for pupils to explain their reasoning to others. Teachers should encourage pupils to both describe how they arrived at their answers and explain why their procedure works. Pupils should be discouraged from simply telling a fellow group member how to answer. Teachers can use these collaborative tasks to support assessment for learning. While these collaborations are designed to work best for pupils working in groups of three, all of the activities may be completed by pairs or individual pupils. Instructions for using the collaborations can be found by clicking the information button (the "i") in each collaboration.
## Overview of the Unit

<table>
<thead>
<tr>
<th>Investigation</th>
<th>Key Mathematical Ideas</th>
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<tbody>
<tr>
<td>Introduction</td>
<td>→ Establish the context for the unit.</td>
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<tr>
<td>Welcome to the English Light Orchestra (10 minutes)</td>
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</table>
| Investigation 1                        | → Figural patterns have repeating structures that can be described using mathematics.  
  Playing with Patterns                   | → Pupils are helped to generalise by examining the structure of figural patterns.  
  (80 minutes)                            | → Pupils calculate the value of the number of lights for a given or ‘fixed’ numerical value and then make this into a variable by ‘unlocking the box’ containing the fixed value. |
| Investigation 2                        | → An unlocked number, or variable, represents the figure number. A figure number is the index for the stage of the pattern displayed.  
  Some Lights are Always On               | → An expression can have multiple terms using the same variable.  
  (55 minutes)                            | → A part of the pattern that does not repeat is represented by a constant; it is added or subtracted in the expression, and does not get multiplied by a variable. |
| Investigation 3                        | → The total number of lights required to make one pattern can be represented by two different expressions which are equivalent, for example: $3n + 4n$ yields the same number of lights as $7n$, for any $n$.  
  Different but the Same                  | → The structure of the pattern can be used to explain why two expressions are equivalent.  
  (55 minutes)                            | → Tables and/or graphs can be used to demonstrate equivalency or non-equivalency.                                                                                                                                 |
| Investigation 4                        | → The additive inverse function is a way of representing subtraction of terms in patterns.  
  It's OK to Be Negative                   | → Equivalent expressions can be created using additive inverses.                                                                                                                                 |
| (65 minutes)                           |                                                                                                                                                                      |
| Collaborations 4.1 and 4.2             | → Equivalent expressions can be justified by colouring the same pattern in different ways.                                                                                                                                 |
| (15 minutes)                           |                                                                                                                                                                      |
| Investigation 5                        | → Equivalent expressions (including negative terms) can be justified by colouring the same pattern in different ways.  
  Your Lighting Design                    |                                                                                                                                                                      |
| (55 minutes)                           |                                                                                                                                                                      |
| Collaborations 5.1 and 5.2             | → Equivalent expressions (including negative terms) can be justified by colouring the same pattern in different ways.                                                                                                                                 |
| (15 minutes)                           |                                                                                                                                                                      |
Key Ideas

→ Establish context for the unit.

Like other Cornerstone Maths units, we use this context lightly, as a grounding story. The context for this unit is the design of large LED lighting displays using figural patterns.

Main Activity

Whole Class | 10 minutes

Read this introduction with the class. Discuss the context of the unit. If you have time, have ask pupils to have a look at the websites mentioned.

Some pupils may have difficulty reading longer texts. Use different reading aloud and group reading techniques to engage all pupils in learning about the context.
Introduction: Welcome to the English Light Orchestra

Light art installation by Yayoi Kusama at Tate Modern.

Our design collective is developing the English Light Orchestra. We aim to delight the whole of England with our lighting displays which combine art, technology and mathematics. We are launching our pilot and soon our displays will light up walls in train stations, sides of buildings and even sides of buses. Our programmers have developed software for us to use in developing the art displays. You will develop our display prototypes using the software and the mathematics of patterns.

Light-emitting diodes (LEDs) are the lights of the future because they produce more light while using less power than incandescent light bulbs. As LED costs have declined, LED installations have grown as an art form and as advertising platforms as well. Artists (e.g., http://tinyurl.com/KusamaAtTateModern) and advertising agencies (e.g., check out http://tinyurl.com/LEDonMini) from around the world have begun exploring this new technology with eye-popping results.
Teacher Notes

Teacher Notes for Investigation 1

Key Ideas

→ Figural patterns have repeating structures that can be described using mathematics.
→ Pupils are helped to generalise by examining the structure of figural patterns.
→ Pupils calculate the value of the number of lights for a given or ‘fixed’ numerical value and then make this a variable by ‘unlocking’ the fixed value.

Start by introducing the pupils to the Pattern Player where figural patterns (collections of images or what we call "figures") are displayed. The figures in the Pattern Player are displayed in random order to scaffold pupils in making sense of the structure in the pattern, rather than counting differences. In this part of the investigation, pupils need only focus on the Pattern Player in the software and draw patterns to match what they see. Introduce the pupil to the required vocabulary of ‘structure’, ‘figure’, ‘pattern’ and ‘generalise’.

In the second part of the investigation, demonstrate how to use the software to create figural patterns in the Designer's Grid and build algebraic expressions. Pupils then try this for themselves using the software. You will guide pupils in learning how to "unlock" numbers to make them variables, a key step in describing how a pattern changes. After pupils have grasped the basic idea, lead a discussion of the process of ‘unlocking a number’ with your pupils and the features of locked and unlocked numbers.

Introducing the software

Useful information

| • A block can only be built using one colour.   |
| • Expressions with only locked numbers represent a particular figure.   |
| • Locked numbers can only be changed, edited, and linked.   |
| • An unlocked number represents a variable.   |
| • Expressions with unlocked numbers (named variables) represent all the possible figures, i.e. the general case.   |

To create a pattern:
1. Drag coloured "lights" of the same colour from the palette into the Designer's Grid to re-create the pattern that is shown in the Pattern Player.
2. Select the lights of the same colour, and select "Block" - this will create a row in the Expression Maker. Note that a block can be made of lights of only a single colour.
3. Select "Pattern" and you will get 4 blocks. Select the blocks and then drag to create the desired pattern.

To build an expression to calculate the lights needed for a pattern:
4. In the Expression Maker, drag and drop numbers from the "Lights in block" and "No. of blocks" areas on the left into the "Expression for the no. of lights" section (the possible target areas will be highlighted in green and change from green to blue when the object can be dropped successfully) and choose an operation. This represents the
expression for the number of lights in that set of blocks.

5. When all the desired blocks and their expressions have been created, click on the icon on the left of the block's row, and drag it down to "Expression for Total".

6. When all the desired blocks and their expressions have been created and dragged into the "Expression for Total", complete the expression by choosing the mathematical operations that relate them (usually addition for our mathematical situations).

Software feedback

Scenario:
Tiles have been dragged into the Designer’s grid in the same orientation as a ‘block’ of the figural pattern shown in the Pattern Player.
Feedback:
• In the pattern player, a correct block is coloured black.

Teaching Notes:

Scenario:
A correct looking ‘block’ has been created in the Designer’s Grid
Feedback:
• The block has a grey interior
• The block is represented in the Expression Builder
• In the pattern player, a correct block is coloured black

Teaching Notes

Scenario:
A correct block in the Designer’s Grid has been used to create a pattern - the no. of blocks defaults to 4.
Feedback:
• The blocks in the pattern all have a grey interior
• The block is represented in the Expression Builder
• In the pattern player, the equivalent number of blocks is coloured black

Teaching Notes
**Scenario:**
The expression is built to give a correct total number of tiles in the pattern.

**Feedback:**
- The blocks in the pattern all have a red interior
- In the pattern player, the equivalent number of blocks are coloured black (and remain black if the pattern player is animated)

**Teaching Notes**

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**Scenario:**
The number of blocks is defined as a variable in the Expression Builder

**Feedback:**
- The blocks in the pattern all have a red interior
- In the pattern player, all of the blocks are coloured black (and they all remain black if the Pattern Player is animated).

**Teaching Notes**

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**Scenario:**
The expression for the block is dragged to the Expression for Total.

**Feedback:**
- The blocks in the pattern all have a red interior
- In the pattern player, all of the blocks are coloured red (and they all remain red if the Pattern Player is animated).

**Teaching Notes**
Question 1

Whole Class or Individual | 10 minute

Model how to play, pause and step through the pattern in the Pattern Player using the software and then ask the pupils to answer questions 1(a) to (e). Guide them through the sketching activities quite quickly. Get pupils to share their answers to questions 1 (c) to 1 (e). Model the answers by using precise vocabulary (Figure, Pattern, Lights) and positional language (directly above, to the right, adjacent).

Discussion

Whole Class or Individual | 10 minute

Point out the different windows in the software: the Pattern Player, Designer's Grid, and Expression Builder.

Task

Whole Class | 10 minutes

Introduce pupils to the software for creating a lighting design by modeling their responses to the questions as they work through Activity 1.2. You may need to convene several short plenaries after pupils have had a chance to work independently. Some pupils may need more support than others.

Display Activity 1.2 to show the different figures of a pattern (of type 3n) displaying randomly in the Pattern Player. Pause a couple times so that both the dynamic and static views inform pupil's understanding of the structure of the pattern.

Create the pattern in the Designer's Grid (see Page 15 for a reminder of the steps).

Question 2(a) to 2(g)

Group | 20 minutes

Let pupils work on their own for a bit. As you circulate, you may find that pupils get tired of trying number after number, seeing that no one number "covers" all figures.

Pupils may make numeric expressions without using a variable or simply place additional blocks of light in the Designer's Grid. Ask pupils to identify what is changing from figure to figure. (The number of blocks changes.) You can suggest to pupils that they try the unlock icon. Ask them what it seems to do. Guide as needed, but try to let pupils work it out on their own. Remind pupils of the goal of building an expression that tells how many lights are needed to build the display and that
will properly colour the pattern in the Pattern Player as they work. Draw on those pupils who have found out how to create a variable to help others. Introduce the language of mathematical expression, variable and constant as you progress through the activities.

**Helpful information:** You have built your pattern in the Designer’s Grid for a fixed number of blocks. If you wish to explore a different number of blocks then you need to ‘unlock’ the box containing the number of blocks. Click on the padlock icon under the number of blocks and change the position of the slider. This will help you build a general expression that will always give you the total number of tiles whatever the number of blocks.

**Question 2(h) Group | 10 minutes**
Ensure that explanations connect the expression to the pattern. Explanations should include something about groupings of 3 in the lights, 3 as the multiplier in the expression. Ask pupils what each number and operation means or refers to. Lead a mini-Plenary after this question, where you ensure that the pupils are developing their understanding of the role and need for variables.

**Question 3 Whole Class | 10 minutes**
In this question, pupils repeat the approach as in question 2 with a pattern of $4n$. For some pupils, this will follow easily from their work on earlier questions. This is a chance for some pupils to catch on to the idea of an unlocked number, or variable, through another go. You may ask some pupils to compare expressions for the two patterns, before the plenary.

**Plenary Whole Class | 10 minutes**
Recap the key vocabulary developed in Investigation 1 and particularly talk about what we understand by a *mathematical expression* using variables. Point out the connection between the expression and the total number of lights in the pattern, and the connection between the expression and the structure of the pattern.

**Pupil Difficulties**
- Pupils may have trouble understanding the linear nature of the patterns, identifying the "block" that repeats, or the difference between adding to the end of a pattern and considering successively longer figures.
- Pupils may want to "get the right answer" the first time instead of actually predicting.
- The idea of a variable may be challenging. The idea of an unlocked number will help.
Investigation 1: Playing with Patterns

It is great to have you as part of our English Light Orchestra Campaign team. As a lighting designer, you need to understand the structure of patterns that you will be making in lighting displays.

Open Activity 1.1 to see the first pattern of lights you will work with. Watch as an image of lights appears, then that image is replaced by another image with a different number of lights. We are going to call each of these images a figure in the repeating sequence of lights. We can think of the figure for each term as a snapshot of the repeating or growing pattern. When you first opened the activity, you saw the figure for a randomly selected term in the sequence. Think about what is the same and different among these figures - try to notice the structure of the pattern.

1) Sketch any three of the figures (images) that you see in the Pattern Player.

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Labelling the patterns

If you pause the Pattern Player and step backwards to the first lighting design, we call this Figure 1 of the pattern.

If you step forwards to the fourth design, this would be Figure 4

b) Now sketch Figure 1, Figure 2, and Figure 4 of the lighting pattern.

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<th>Figure 1</th>
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<td>![Figure 1 Image]</td>
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<td>Figure 2</td>
</tr>
<tr>
<td>![Figure 2 Image]</td>
</tr>
<tr>
<td>Figure 4</td>
</tr>
<tr>
<td>![Figure 4 Image]</td>
</tr>
</tbody>
</table>

c) Describe the pattern: What does each of the figures (images) of the pattern have in common?

Answers vary:
They each look like a flower repeating. Each red center is surrounded by 4 green lights. It looks like a cross.
Accept all types of answers, but encourage pupils to say how many lights of each color are needed as well as describe their location.

d) What is changing from one figure to another?

There is a basic shape that is the same, but it was repeated a different number of times.
e) Explain how to calculate the number of lights needed for any of the figures that has randomly appeared on the screen.

At this stage, pupils may give an example rather than a generalisation. However, pupil answers should reflect that there are 5 lights (4 green lights and 1 red light) in every repetition. So to calculate the number of lights needed for a figure with three flowers, for example, we need to multiply $3 \times 5$ to get the total of 15 and for the figure with 5 flowers we need to multiply $5 \times 5$ for a total of 25.

Challenge

Use your imagination to think of a different repeating lighting pattern. Sketch Figure 1 and the Figure 3 of your made-up pattern in the grid below.

**Figure 1**

**Figure 3**

Look for consistent patterns—repeating sets of lights. Figure 1 should have just one of the sets; figure 3 should have three of the sets.

Task

For each pattern in Activity 1.2 and Activity 1.3, you will learn to recreate the pattern in the Designer’s Grid and then build a mathematical expression that tells you how many lights are needed for a display of that pattern, for any figure in the sequence. If you are successful in building a correct mathematical expression, the pattern in the Pattern Player will be lit with the colour you have chosen in the Designer’s Grid.

Open Activity 1.2.

Watch figures display randomly in the Pattern Player. Pause the action a few times to help you notice the pattern’s structure.
2)

a) Describe the pattern’s structure in words or pictures. It might help if you think about where the pattern starts and how it grows. (You may wish to use the Step Forward and Step Back to help).

Answers will vary.
Pupils may say they see an inverted L-shape or an arrow.
It has repeated blocks of three lights.

b) Sketch the building block for the pattern.

![Building block sketch]

b) Sketch the building block for the pattern.

![Building block sketch]


c) Explore how you can recreate the pattern.

d) Now learn to use the designer’s tools to recreate a pattern that is identical to that in the Pattern Player? You will need to follow some simple steps:

i) Use one colour to create the building block for the pattern.

ii) Use the Block and Pattern buttons to recreate the pattern.

iii) You should now see 4 blocks of your pattern and something in the Expression Builder that relates to your pattern.

iv) Drag the icon for your block to the Expression for Total row.

v) Edit the Number of Blocks to give you a different figure.

vi) Construct your expression for the total number of lights for your particular figure in the Pattern Player.

<table>
<thead>
<tr>
<th>Expression for Total</th>
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</thead>
<tbody>
<tr>
<td>The Expression for Total tells you how many lights are needed for your pattern, no matter what figure is shown.</td>
</tr>
</tbody>
</table>


e) Edit the expression in the Expression Builder to give you the total number of lights for any figure, no matter which figure is shown. If you are successful, the lights in the Pattern Player will be the same colour as the lights in the Designer’s Grid.
This is where it is important to let the pupils struggle productively.
Point out the hint first and let them have a go. Only after the pupils struggle for a bit should you reveal the need for a variable.
Correct expressions are of the form $3 \times \text{Name}$ (whatever name they choose for the variable) and it will colour the pattern no matter what the figure number.
Suggest pupils use the slider on the variable to watch how the pattern in the Designer's Grid changes as the numbers for the variable changed.

f) Select the name of your variable and drag the slider that appears to check if your pattern grows as expected.

![Variable slider](image)

Correct expressions are of the form $3 \times \text{Name}$ (whatever name they choose for the variable) and it will colour the pattern no matter what the figure number.
Suggest pupils use the slider on the variable to watch how the pattern in the Designer's Grid changes as the numbers for the variable changed.

h) In your expression:

i) What does the number represent?

The ‘3’ represents the number of lights in a block.

ii) Which part is the variable and what does it represent?

The variable is the name or letter given to the unlocked number.
It represents the number of blocks (which can vary).

Open Activity 1.3. Watch the figures display randomly in the Pattern Player. Pause the action a few times to help you notice the pattern’s structure.

3)

a) Describe the pattern’s structure in words or pictures. It might help if you think about where the pattern starts and how it grows. (You may wish to use the Step Forward and Step Back to help).

Answers will vary.
Pupils may say they see steps or staircases.
It has repeated blocks of four lights.

b) Sketch the building block for the pattern.

![Building Block for Pattern]

(b) Sketch the building block for the pattern.

c) Recreate the identical pattern in the Pattern Player. Here is a reminder of the steps:

i) Use one colour to create the building block for the pattern.
ii) Use the Block and Pattern buttons to recreate the pattern.
iii) You should now see something in the Expression Builder that relates to your pattern.
iv) Edit your expression so that it gives you the total number of lights for your particular figure in the Pattern Player.
v) Drag the icon for your block to the Expression for Total row.
vi) Edit the expression in the Expression Builder to give you the total number of lights for any figure, no matter which figure is shown. If you are successful, the lights in the Pattern Player will be the same colour as the lights in the Designer's Grid. Use the slider to check.

Hint: Click the padlock icon under the "No. of blocks".

d) Select the name of your variable and drag the slider that appears to check if your pattern grows as expected.

e) Copy your Expression for Total.

Correct expressions are of the form $4 \times \text{Name}$ (whatever name they choose for the variable) and colour the pattern no matter what the figure number.

Suggest pupils use the slider on the variable to watch how the pattern in the Designer's Grid changes as the numbers for the variable changed.

f) In your expression:

i) What does the number represent?
ii) Which part is the variable and what does it represent?

The variable is the name or letter given to the unlocked number. It represents the number of blocks (which can vary).
Teacher Notes for Investigation 2: Some Lights are Always On

Key Ideas

→ An unlocked number, or variable, represents the figure number. A figure number is the index for the stage of the pattern displayed.
→ An expression can have multiple terms using the same variable.
→ A part of the pattern that does not repeat is represented by a constant; it is added or subtracted in the expression.

55 minutes total

Question 1

Group | 15 minutes
Pupils should be told to build the pattern using two different coloured blocks of lights. When they unlock, they will usually choose different variable names for the two parts of the pattern. They should be encouraged to use the slider so they can see that there is a need to link the variables in some way.
Help pupils to think of a way to synchronise the animation by ‘linking’ their variables. Let them come up with methods to try. There are two ways to do this:
1. Give both variables the same name.
2. Drag the name of one variable onto another, which has the same effect.

A software warning message alerts pupils when they are about to link variables. Encourage them to read and understand this message.

Software suggestions:

• If a pattern is made from blocks of different coloured lights, each block will need to be created separately.
• A constant term in a pattern is represented by a group of lights that are defined as a block but not made into a pattern.
• Pay attention to the direction in which the pattern grows. When creating a pattern you should drag the last light in this direction.

Question 2

Group | 15 minutes
This pattern has three different sets of coloured lights, two of them (green and orange) vary and one (purple) is constant. Let pupils try this activity on their own for 5 minutes and share their suggestions for how to build the expression for the purple lights.

Question 3

Group | 15 minutes
This pattern has four different sets of coloured lights, three of them (blue, red and green) vary and one (yellow) is constant. Let pupils try this activity on their own for 5 minutes and share their suggestions for how to build the expression for the total number of lights.

Plenary

Whole Class | 10 minutes
Encourage pupils to share with each other the issues and insights that came up during the activities. Ask which was the most challenging pattern and why; likewise, which was easiest.
The software gives pupils feedback that they have an expression that represents the pattern in the Designer’s Grid.

Ask how else they know they have the right expression.

Points to make:
- Highlight the vocabulary "variable" and "constant" within the context of the lighting patterns.
- Variables are linked when they need to represent the same unlocked number.
- Constants are parts of the expression (and pattern) that do not change.

**Pupil Difficulties**
- Pupils may unlock numbers and give them different names. But variables that represent the same thing need to have the same name or the pattern won colour properly. For example, pupils may assign each colour in a pattern a different variable. Using the names of colors as the names for their variables can lead to this.
- Pupils may try to add constants and variables together. i.e. \((3 \times n) + 1 = 4n\)
Investigation 2: Some Lights are Always On

The person who has commissioned the lighting design is visiting the office. We need to show her that our lighting designers are learning the software according to the project schedule. We are meant to show off three kinds of patterns by the end of today. Also, don’t forget that she might ask you to explain how you did it.

Try matching the different lighting patterns. You will need to build an expression for how many lights it takes to make any figure in the patterns.

Open Activity 2.1.
Watch the figures display randomly in the Pattern Player. Pause the action a few times to help you notice the pattern’s structure.

1) a) Describe the pattern’s structure in words or pictures and how it is different from other patterns you have seen. It might help if you think about the starting pattern and how it grows. (You may wish to use the Step Forward and Step Back to help).

Accept multiple answers.
The pattern looks like a tree. It has two colours. There are 5 green lights and 3 orange ones that repeat.

b) Predict: What do you think the expression to give you the total number of lights (for any pattern of this type) will be?

Accept all predictions. Pupils may say \(5 \times \) green + \(3 \times \) orange.
c) Sketch the building block for the pattern.

In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern. You will need to build the blocks for each colour separately. Use the slider to check. You may want to look back at Investigation 1 to remind yourself how to do this.

Most pupils will create expressions such as: \((5 \times \text{green}) + (3 \times \text{orange})\) that include two different variables. Learning about the need to link variables is the goal of the activity.

d) **Check**: Was your prediction correct? [The lights in the Pattern Player would be correctly coloured]. If not, modify your expression.

Most pupils will create expressions such as: \((5 \times \text{green}) + (3 \times \text{orange})\) that include two different variables. If the different coloured blocks do not animate together, there is a problem. It is likely that pupils have named the variables for each block differently. They need to "link" the two variables (by naming them the same) to make the Pattern Player work the way they want. Learning about the need to link variables is the goal of the activity. Correct expressions will be of the type \((5 \times \text{tree}) + (3 \times \text{tree})\).

e) **Explain**: How the numbers and variables in your expression are related to the lights in the pattern.

The number 5 relates to the number of lights in a block of green lights. The number 3 relates to the number of lights in a block of orange lights. The variable (unlocked number) tells you what figure number you are on, or how many times to repeat the pattern.
Open Activity 2.2. Watch the figures display randomly in the Pattern Player. Pause the action a few times to help you notice the pattern's structure.

2)
   a) Sketch the first two figures for the pattern.

   ![Figure 1](image1)
   ![Figure 2](image2)

   b) Describe the pattern's structure in words and how it is different from other patterns you have seen. It might help if you think about the starting pattern and how it grows. (You may wish to use the Step Forward and Step Back to help).

   Answers will vary. This pattern has the same structure as before (a tree) but it also has 4 purple lights that do not repeat - a constant term.

   NOTE: A constant term is a group of lights that are made into a ‘block’ – but not then made into a pattern.

   c) **Predict**: What is the expression for the total number of lights?

   Correct expressions will look like $(5 \times \text{tree}) + (3 \times \text{tree}) + 4$
   However, as this pattern looks similar to the previous one but has an extra block of 4 purple lights, pupils may create an expression like $(5 \times \text{tree}) + (3 \times \text{tree}) + (4 \times \text{purple})$ – i.e. creating a variable term for the purple lights instead of a constant.
In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern. Use the slider to check.

d) **Check**: Was your prediction correct? [The lights in the Pattern Player would be correctly coloured]. If not, modify your expression.

Pupils may not know how to deal with the part of the pattern that does not repeat.
One possible problem is that pupils have made a repeating block (or pattern) for the constant.

e) **Explain**: How are the numbers and variables in your expression related to the lights in the pattern? Make a sketch if it helps.

The number 5 relates to the number of lights in a block of green lights.
The number 3 relates to the number of lights in a block of orange lights.
The variable (unlocked number) tells you what figure number you are on, or how many times to repeat the pattern.
The 4 relates to the 4 purple lights that are constant.
Challenge

Open Activity 2.3. Look at the pattern in the Pattern Player. What, if anything, is different about this pattern than those you have seen before?

3)
   a) Describe the pattern's structure in words or pictures.

   Accept multiple answers.
   The pattern "grows" in three different directions (this does not matter mathematically).
   The blue, red and green light patterns grow from the middle outwards.
   There is one yellow light that is constant in the pattern.

   b) Sketch the first two figures so that you can identify the building block for the pattern.

   ![Figure 1]
   ![Figure 2]

   c) **Predict**: What is the expression for the total number of lights? Record your prediction.

   Accept any predictions. The pattern has a centre (yellow) light that remains constant, while the three blocks of different coloured lights vary together. Expressions might be: $(1 \times n) + (1 \times n) + (1 \times n) + 1$ or $(1 \times \text{red}) + (1 \times \text{blue}) + (1 \times \text{green}) + 1$

In the Designer's Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern.
Use the slider to check.

d) **Check**: Was your prediction correct? [The lights in the Pattern Player would be correctly coloured]. If not, modify your expression.

e) **Explain**: How the numbers and variables in your expression are related to the lights in the pattern

The expression \((3 \times n) + 1\) (or the equivalent \((1 \times n) + (1 \times n) + (1 \times n) + 1\)) indicates that three blocks are varying (growing) together while the centre remains constant.
Teacher Notes

Teacher Notes for Investigation 3

Key Ideas

→ The total number of lights required to make one pattern can be represented by different expressions which are equivalent, for example: $3n + 4n$ is an expression for the same total number of lights as $7n$, for any $n$.

→ The structure of the pattern can be used to explain why two expressions are equivalent.

→ Tables and/or graphs can be used to justify equivalency.

Pupils will need to compare at least two colourings/expressions for the same pattern using the tabs. The patterns are: Columns, Train tracks and Bridges, which lead to different types of algebraic expressions. They are presented without colour in the Pattern Player so that pupils can choose their own colours according to how they see the pattern. This will result in a diversity of algebraic expressions.

<table>
<thead>
<tr>
<th>Activity 3.1</th>
<th>Activity 3.1</th>
<th>Activity 3.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Columns</td>
<td>Train tracks</td>
<td>Bridges</td>
</tr>
</tbody>
</table>

A correct "colouring" uses the coloured lights to model the pattern exactly as in the Pattern Player.

55 minutes total

Question 1
Pupils work in their groups to match a pattern from the Pattern Player. The pattern does not involve a constant term but gives an opportunity to develop pupils’ understanding of equivalence for expressions of the type $a \times n$.

or

Question 2
Pupils work in their groups to match a pattern from the Pattern Player. The pattern involves a constant term and gives an opportunity to develop pupils’ understanding of equivalence for expressions of the type $(a \times n) + c$.

Challenge
Pupils work in their groups to match a pattern from the Pattern Player. The pattern involves a constant term and gives an opportunity to develop pupils’ understanding of equivalence for expressions of the type $(a \times n) + c$.

Plenary
10 minutes total
Invite pupils to show their different expressions for the same pattern. Lead a plenary where pupils share their explanations for why their expressions are (or are not) equivalent. Encourage the use of the Table and the Compare functionality to highlight how the different explanations are supported.
or not.
Introduce the term equivalent expressions.
Equivalent expressions give the same number of total lights needed to build a particular figure in the same pattern – but coloured in different ways.

**Pupil Difficulties**
Pupils may have difficulty seeing a pattern in more than one way once they have understood the pattern in a certain way (like when seeing an optical illusion). Pupils may decide that the first Figure is in fact a constant, which will lead to conflicting expressions that they may have difficulty explaining.
Investigation 3: Different but the Same

We have a new contract for three more art displays. We have three different ideas for lighting patterns and you can choose to include several colours. We need to be able to calculate how many lights to buy for each pattern. We don’t yet know how much space we will get at each location so mathematical expressions will be helpful.

In this investigation, you will decide how to colour the patterns using as many colours as you like. Then you will compare the expressions for these colourings. You will learn how to use tables and graphs to support your thinking.

Open Activity 3.1. Watch how the display of lights changes in order to understand its structure.

In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern. Use your slider to check.

1)
   a) Copy your lighting design and its Expression for Total below.

Patterns should demonstrate that pupils have a sense of how the colouring works.
Some sample expressions are:
8n
or 4n + 4n or
2n + 2n + 2n + 2n
i.e. Pupils might create a pattern represented by 4n + 4n as shown by the building block below.
Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.

b) Copy your table.

[Use as many columns as you need]

Pupils’ answers will vary.
The expressions for the total number of lights should be equivalent to $8n$
i.e. if the pupils created a pattern represented by $4n + 4n$, they might produce a table as shown below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$4n$</th>
<th>$4n$</th>
<th>Total</th>
</tr>
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<tbody>
<tr>
<td>4</td>
<td>16</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>7</td>
<td>28</td>
<td>28</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

Can you see the pattern differently? Go to Tab 2 and recreate the pattern using a different lighting design. Use the software to build an expression to calculate the total number of lights for any figure in the pattern.

c) Copy your lighting design and its Expression for Total.

Encourage pupils to "look at" the pattern differently. You may need to help some pupils by giving them an idea, such as: "In your last pattern you grouped these lights together, how about grouping these other lights together?"
i.e. Pupils might see the pattern as $2n + 2n + 2n + 2n$ as shown below.

Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.
d) Copy your table below.
   [Use as many columns as you need]

Pupils’ answers will vary.
The expressions for the total number of lights should be equivalent to $8n$
i.e. if the pupils created a pattern represented by $2n + 2n + 2n + 2n$ as shown previously, they might produce a table as shown below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$2 \times n$</th>
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<td>8</td>
<td>8</td>
<td>32</td>
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</tbody>
</table>

Compare your two expressions.

e) What do you notice? Explain how you know that the two expressions are equivalent (or not).

There should be a verbal description of the expression for the total number of lights, e.g., the variable number times 8 equals the total number of lights.

Compare your two tables of values.

f) What do you notice?

Pupils should comment on the multiplicative relationships that they see in the table. If their two tables contained the same values of $n$, they might notice that for the same values of $n$, the total number of lights are equal.

Make a graph by creating a graph snapshot.

g) What do you notice about your graph??

Pupils might comment that the coordinate points are in a line.

2) Open Activity 3.2. Watch how the display of lights changes in order to understand its structure.

In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to
build an expression to calculate the total number of lights for any figure in the pattern. Use your slider to check.

a) Copy your lighting design and its Expression for Total below.

Patterns should demonstrate that pupils have a sense of how the colouring works.
Some sample expressions are:
7n + 2
or 6n + 1n + 2
or 5n +2(n+1)
or other equivalent expressions.

Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.

b) Copy your table below.

[Use as many columns as you need]

Pupils’ answers will vary.
The expressions for the total number of lights should be equivalent to 7n + 2

Can you see the pattern differently? Go to Tab 2 and recreate the pattern using a different lighting design. Use the software to build an expression to calculate the total number of lights for any figure in the pattern.

Encourage pupils to "look at" the pattern differently. You may need to help some pupils by giving them an idea, such as: "In your last pattern you grouped these lights together, how about grouping these other lights together?"

c) Copy your lighting design and its Expression for Total below.
Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.

d) Copy your table below.

[Use as many columns as you need]

Compare your two expressions

e) What do you notice? Explain how you know that the two expressions are equivalent (or not).

There should be a verbal description that the two expressions are equivalent because they have they can be simplified to equal \(7n + 2\).

Compare your two tables of values.

f) What do you notice?

Pupils should comment on the multiplicative relationships that they see in the table.
If their two tables contained the same values of \( n \), they might notice that for the same values of \( n \), the total number of lights are equal.

Make a graph by creating a graph snapshot.

\[
g) \quad \text{What do you notice about your graph??}
\]

Pupils might comment that the coordinate points are in a line.
**Challenge**

Open Activity 3.3. Watch how the display of lights changes in order to understand its structure.

In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern. Use your slider to check.

a) Copy your lighting design and its **Expression for Total** below.

Patterns should demonstrate that pupils have a sense of how the colouring works.
Some sample expressions are:
- $6n + 3$
- $5n + 1n + 2$
- $3n + 3(n+1)$
- or other equivalent expressions.

Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.

b) Copy your table below.

[Use as many columns as you need]

Pupils’ answers will vary.
The expressions for the total number of lights should be equivalent to $6n + 3$
Can you see the pattern differently? Go to Tab 2 and recreate the pattern using a different lighting design. Use the software to build an expression to calculate the total number of lights for any figure in the pattern.

c) Copy your lighting design and its **Expression for Total** below.

Pupils’ answers will vary.
The expressions for the total number of lights should be equivalent to $6n + 3$

Use the slider to change the value of your variable. Create a table snapshot. Do this a few more times to create several rows in your table.

d) Copy your table below.

[Use as many columns as you need]

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Compare your two expressions.

e) What do you notice? Explain how you know that the two expressions are equivalent (or not).

There should be a verbal description that the two expressions are equivalent because they have they can be simplified to equal $6n + 3$.

Compare your two tables of values.

f) What do you notice?
Pupils should comment on the multiplicative relationships that they see in the table. If their two tables contained the same values of n, they might notice that for the same values of n, the total number of lights are equal.

Make a graph by creating a graph snapshot.

\[ \text{g) For each coordinate point, what is the relationship between the value of the variable and the total number of lights needed?} \]

Pupils might comment that the coordinate points are in a line.
Teacher Notes

Teacher Notes for Investigation 4

Key Ideas

→ The additive inverse function is a way of representing subtraction of terms in patterns.
→ Equivalent expressions can be created using additive inverses.

Discussion

Whole Class | 5 minutes

65 minutes total

In our software, the negative lights are an additive inverse. This investigation leads pupils to realise that a term with an additive inverse can be used to build non-identical equivalent expressions that produce the same pattern.

Question 1

Whole Class | 10 minutes

As a class, read the context. Give groups five minutes to try to come up with a solution.

Lead the class in brief discussion of negative lights and their interaction with positive lights and their numeric value with key points as follows:

• Make the point that subtraction is the inverse of addition. If we can add something, we can also add its inverse to get back to our starting place. i.e. \(5 - 3 = 5 + (-3)\)
• Tell the class that we can do the same with our lights and blocks using a negative light, \(\times\).

Demonstrate to pupils that in the Designer's Grid a positive light overlaid by a negative light results in no light on that grid cell. An example demonstration could proceed as follows:

Drag 2 lights out onto the Designer's Grid.

Ask: How can we end up with 5 lights? Add three more lights.
How do we get back to 2? We could subtract 3 lights. But we could also (prompt pupils towards "add negative 3").
Point out the negative light in the Designer's Grid palette, \(\times\).
What should happen when I put a negative block onto a positive block? (get the pupils to think about what should happen – how to represent a positive light co-located with a negative light)

Place a negative light onto the same cell where a positive light is and ask what the value of the cell is, ensuring that pupils understand that the cell is empty because the value is zero. Note that when the positive light is already in a block, the negative light only interacts with the positive block when the negative light is made into a block as well.

Question 2

Group | 20 minutes

Encourage the pupils to step backwards and forwards to identify the building block for the pattern – and then to imagine which light may have been switched off by building a block of negative lights.

Discussion

Whole Class | 10 minutes

Lead a discussion about pupils' answers to question 6. Points to make:

• A pattern can be "visualized" in different ways.
• Equivalent expressions can be reduced to the same simplest form (i.e., \((a \times n) + c\) or \((a \times n) - c\))

**Pupil Difficulties**
Pupils may not understand subtraction as the inverse of addition.
Pupils may have difficulty "seeing" the overlapping lights in the patterns.
Investigation 4: It’s OK to Be Negative

A new art display is going into Sheffield station. Because it is sponsored by the Le Lion clothing brand, whose brand logo is a simple block letter L, one of the patterns to be displayed has to be a series of L shapes. The brand’s owner, Julia Lettiger, promised her two children that they could help make the art for the pattern. Each child made one block.

Ms. Lettiger has given two constraints: 1) Each of the L shapes must be exactly four lights tall and four lights wide; and 2) both of the children’s blocks must be used in the pattern. Our programmers have already put the children’s blocks into our software.

Use Activity 4.1 to create the L-shaped pattern using the two blocks provided and build the matching expression.

[HINT: You will need to learn how to use the ✗ light to switch off coloured lights].

1)
   a) Copy the expression.

Answers should reflect the need for a ‘negative light’. A correct expression might be: \((8 \times n) - n\).

b) Describe how your expression represents the L-shaped pattern.

Answers should explain the use of the negative lights to subtract the lights that are not needed as they are counted twice.
Our designer Samuel has created a new design.

Open Activity 4.2. Samuel says he built the pattern using one block of two negative lights. He didn't say what the rest of the blocks were.
You need to create a pattern and expression that matches Samuel's.

2)  
   a) Describe the pattern’s structure in words and pictures.

   The block looks like a letter ‘H’
   There are two columns of three red lights with one blue light between them.
   BUT there are two ‘hidden’ lights in each block.

   b) Predict: What do you think the expression to give you the total number of lights will be?

   The expression for the total number of lights should simplify to \( 7n \) because each block has 7 repeating lights and there is no constant term.

In the Designer’s Grid, make a pattern to match the Pattern Player. Use the software to build an expression to calculate the total number of lights for any figure in the pattern. Use your slider to check.

   c) Check your prediction by building the expressions. Did your prediction work?

   If not, revise your expression and Copy your new prediction.

   d) Try to build the pattern in the Designer’s Grid making use of a block of negative lights.

   e) Predict: Write an expression for the pattern shown.
Answers will vary. Some examples are: $(6 \times n) + (3 \times n) + (-2 \times n)$, $(9 \times n)$- $(2 \times n)$, and incorrectly $(3 \times n) + (1 \times n) + (1 \times n) + (1 \times n) + (1 \times n)$. Answers should include the addition of a negative.

f) Check your prediction by building the expressions. Did your prediction work? If not, revise your expression and write down the correct one below.

3) Now try making the pattern without using negative lights.

   a) Build the pattern in a new tab.

   b) Predict: Write an expression for the pattern shown.

   Sample answer: $(7 \times n)$

   c) Check your prediction by building the expressions. Did your prediction work? If not, revise your expression and write down the correct one below.

4) Compare the two patterns and expressions. Explain how you know they are equivalent.

The two patterns should look identical.
Both expressions should simplify to give $(7 \times n)$. 
Teacher Notes
Teacher Notes for Collaborations 4.1 and 4.2

Key Ideas

→ Different equivalent expressions can be generated for the same figural pattern.
→ Collecting like terms can be used to justify equivalent expressions.

Main Activity 15 minutes total

Each pupil should identify one correct expression by dragging their bar over it. Encourage pupils to justify their choices by copying the pattern onto squared paper and showing how it might be coloured.
Teacher Notes
Teacher Notes for Investigation 5

Key Ideas

→ Different patterns can be represented by the same expression, of the form \((a \times n) + (b \times n) + c\)

Pupils have: (1) replicated a coloured pattern by making an expression; (2) coloured a pattern to make an expression and compared it with others to see how the same pattern can be represented by different expressions.

Now they will make their own pattern, or lighting design, that can be represented by a given expression. By comparing their designs, pupils can see how very different patterns can be associated with the same expression through the quantities of the colours used. Note that by \((7 \times n) + 2\), we mean all expressions equivalent to it.

55 minutes total

Questions 1 (a) – 1 (e)

Group | 15 minutes

By using different colours, pupils can make expressions that are equivalent to \((7 \times n) + 2\). Help them see how each \(n\) term contributes to \(7 \times n\) and how 2 is not repeated (it is a constant). This should be somewhat familiar now from other activities, so you can be more explicit about how, say, \((5 \times n) + (2 \times n)\) is equivalent to \((7 \times n)\).

Questions 1 (f) – 1 (g)

Group | 15 minutes

Ask pupils to compare patterns. They may do this in a "gallery walk" where pupils circulate to visit each computer where the display shows the unique pattern. Challenge them to find patterns that used the same number of reds, but in a very different shape, for example. Reinforce that even though the patterns look quite different, the same total number of lights are needed for the display. You could also have pupils figure out how many lights are needed for all their displays together, helping them see the \((7 \times n) + 2\) structure in that calculation.

Discussion

Whole Class | 10 minutes

Lead a discussion. Highlight the following:

• Unlocked numbers are used to represent varying quantities.
• A pattern can be represented with an expression relating the figure number to the total number of lights to build that figure.
• Equivalent expressions each give the accurate total number of lights, even though they "look" different.

Final task

Group | 10 minutes

Invite pupils to share their final conclusions about what they have learned during the unit. Bring up any rules that pupils have developed for adding like terms and relating equivalent expressions. Connect the module to future learning (e.g., algebraic notation).
Investigation 5: Your Lighting Design

Now it is your turn to make your own lighting design art piece to add to the Electric Light Orchestra.
The challenge is to make up a unique pattern that can be expressed as \((7 \times n) + 2\).

1.
   a) Sketch a pattern for which the total number of lights for any figure can be found with the expression \((7 \times n) + 2\). You may want to start with one colour only.

Make this pattern in the Designers Grid and build an expression for the total number of lights to match the pattern.

   b) Copy your expression for the total number of lights.

   c) Explain how you know your pattern can be expressed by \((7 \times n) + 2\).

Answers will vary. Some possibilities: 7 of my lights are repeated every time, and 2 are not. Or, my repeating blocks of 3 blues and 4 reds make 7 lights in my pattern, which gets repeated. The two does not repeat.

Once built in the real world, your lighting display will flash random figures from your pattern, just as the Pattern Player does. The largest figure shown will be figure number 100.

   d) Use the table, graph and expression to find the total number of lights you will need for your pattern — how many of each colour. Explain your reasoning.

Answers will vary given the pupils’ designs, but all answers should show how figure number 100 will take 702 lights. Some pupils may show this symbolically (e.g. For figure 100, there will be 100 red blocks of 3 each and 100 green blocks of 4 each. that is 100 times 3 plus 100 times four. Then I add on 2, for a total of 702) and others may reason from the table (e.g., since \(10 \times 7\) is 70, then \(100 \times 7\) is 700) or graph.
If pupils say they need numbers other than 702, ask them to explain their reasoning and help them sort out their mistake.

Now create another pattern for which the total number of lights for any figure can be found with the expression \((7 \times n) + 2\) using as many colours as you like. Try to make your pattern unique!

Look at another group's or pupil's pattern.

e) Explain how their design could be expressed by \((7 \times n) + 2\).

Pupils should notice that regardless of the pattern, the total number of lights for a given figure number is the same.

f) Decide if the total expression for your pattern is equivalent to the total expression for their pattern. Show how they are equivalent or not.

By now pupils should be able to add and subtract like terms and show how each expression is equivalent to \(7n + 2\).

g) How could you tell that \((7 \times n) + 2\) could express both their pattern and your pattern?

Pupils' responses may vary. For example: Even though they look very different, I could see that groups of 7 lights were switching on and off (repeating) and 2 lights that always stay lit.

Final task

Our biggest supporter has arranged a surprise launch party for the campaign. She tells us that the partygoers will want a brief explanation of the work that we do to prepare the art displays.

Explain how a figural pattern may be described by a mathematical expression. Provide an example if it would be helpful to describe the relationship between the expression and the pattern.
Key Ideas

→ Different equivalent expressions can be generated for the same figural pattern.
→ Collecting like terms can be used to justify equivalent expressions.

Main Activity 15 minutes total

Each pupil should contribute to identifying two sets of equivalent expressions. Encourage pupils to justify their choices by copying the pattern onto squared paper and showing how it might be coloured.