Teaching for Mastery
Questions, tasks and activities to support assessment

Year 5
Mike Askew, Sarah Bishop, Clare Christie, Sarah Eaton, Pete Griffin and Debbie Morgan
About the authors 3
Introduction 4
The structure of the materials 8
Number and Place Value 9
Addition and Subtraction 11
Multiplication and Division 14
Fractions 17
Measurement 21
Geometry 25
Statistics 28

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About the authors

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Pete Griffin works at a national level as Assistant Director for the National Centre for Excellence in the Teaching of Mathematics. Pete has experience as a secondary teacher, Advisory Teacher, and lecturer in Mathematics Education at the Open University. Pete has worked with QCA and the National Strategies and has written and developed a wide range of teacher professional development materials.

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Clare Christie is a primary teacher and Maths Leader. Clare is also a Mathematics SLE, supporting schools with Maths teaching and learning. Clare is primary lead of the Boolean Maths Hub and a member of the ACME Outer Circle.
Introduction

In line with the curricula of many high performing jurisdictions, the National curriculum emphasises the importance of all pupils mastering the content taught each year and discourages the acceleration of pupils into content from subsequent years.

The current National curriculum document¹ says:

‘The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils’ understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.’ (National curriculum page 3)

Progress in mathematics learning each year should be assessed according to the extent to which pupils are gaining a deep understanding of the content taught for that year, resulting in sustainable knowledge and skills. Key measures of this are the abilities to reason mathematically and to solve increasingly complex problems, doing so with fluency, as described in the aims of the National curriculum:

‘The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.’ (National curriculum page 3)

Assessment arrangements must complement the curriculum and so need to mirror these principles and offer a structure for assessing pupils’ progress in developing mastery of the content laid out for each year. Schools, however, are only ‘required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study’ (National curriculum page 4). Schools should identify when they will teach the programmes of study and set out their school curriculum for mathematics on a year-by-year basis. The materials in this document reflect the arrangement of content as laid out in the National curriculum document (September 2013).

These Teaching for Mastery: Questions, tasks and activities to support assessment outline the key mathematical skills and concepts within each yearly programme and give examples of questions, tasks and practical classroom activities which support teaching, learning and assessment. The activities offered are not intended to address each and every programme of study statement in the National curriculum. Rather, they aim to highlight the key themes and big ideas for each year.

¹. Mathematics programmes of study: key stages 1 and 2, National curriculum in England, September 2013, p3
Ongoing assessment as an integral part of teaching

The questions, tasks, and activities that are offered in the materials are intended to be a useful vehicle for assessing whether pupils have mastered the mathematics taught.

However, the best forms of ongoing, formative assessment arise from well-structured classroom activities involving interaction and dialogue (between teacher and pupils, and between pupils themselves). The materials are not intended to be used as a set of written test questions which the pupils answer in silence. They are offered to indicate valuable learning activities to be used as an integral part of teaching, providing rich and meaningful assessment information concerning what pupils know, understand and can do.

The tasks and activities need not necessarily be offered to pupils in written form. They may be presented orally, using equipment and/or as part of a group activity. The encouragement of discussion, debate and the sharing of ideas and strategies will often add to both the quality of the assessment information gained and the richness of the teaching and learning situation.

What do we mean by mastery?

The essential idea behind mastery is that all children need a deep understanding of the mathematics they are learning so that:

- future mathematical learning is built on solid foundations which do not need to be re-taught;
- there is no need for separate catch-up programmes due to some children falling behind;
- children who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

There are generally four ways in which the term mastery is being used in the current debate about raising standards in mathematics:

1. A mastery approach: a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time. Pupils are neither ‘born with the maths gene’ nor ‘just no good at maths’. With good teaching, appropriate resources, effort and a ‘can do’ attitude all children can achieve in and enjoy mathematics.

2. A mastery curriculum: one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and ideas and to the rich connections between them. There is no such thing as ‘special needs mathematics’ or ‘gifted and talented mathematics’. Mathematics is mathematics and the key ideas and building blocks are important for everyone.

3. Teaching for mastery: a set of pedagogic practices that keep the class working together on the same topic, whilst at the same time addressing the need for all pupils to master the curriculum and for some to gain greater depth of proficiency and understanding. Challenge is provided by going deeper rather than accelerating into new
mastery. Teaching is focused, rigorous and thorough, to ensure that learning is sufficiently embedded and sustainable over time. Long term gaps in learning are prevented through speedy teacher intervention. More time is spent on teaching topics to allow for the development of depth and sufficient practice to embed learning. Carefully crafted lesson design provides a scaffolded, conceptual journey through the mathematics, engaging pupils in reasoning and the development of mathematical thinking.

4. Achieving mastery of particular topics and areas of mathematics. Mastery is not just being able to memorise key facts and procedures and answer test questions accurately and quickly. It involves knowing ‘why’ as well as knowing ‘that’ and knowing ‘how’. It means being able to use one’s knowledge appropriately, flexibly and creatively and to apply it in new and unfamiliar situations. The materials provided seek to exemplify the types of skills, knowledge and understanding necessary for pupils to make good and sustainable progress in mastering the primary mathematics curriculum.

Mastery and the learning journey

Mastery of mathematics is not a fixed state but a continuum. At each stage of learning, pupils should acquire and demonstrate sufficient grasp of the mathematics relevant to their year group, so that their learning is sustainable over time and can be built upon in subsequent years. This requires development of depth through looking at concepts in detail using a variety of representations and contexts and committing key facts, such as number bonds and times tables, to memory.

Mastery of facts, procedures and concepts needs time: time to explore the concept in detail and time to allow for sufficient practice to develop fluency.

Practice is most effective when it is intelligent practice, i.e. where the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity. (Gu 2004) The examples provided in the materials seek to exemplify this type of practice.

Mastery and mastery with greater depth

Integral to mastery of the curriculum is the development of deep rather than superficial conceptual understanding. The research for the review of the National Curriculum showed that it should focus on “fewer things in greater depth”, in secure learning which persists, rather than relentless, over-rapid progression. It is inevitable that some pupils will grasp concepts more rapidly than others and will need to be stimulated and challenged to ensure continued progression. However, research indicates that these pupils benefit more from enrichment and deepening of content, rather than acceleration into new content. Acceleration is likely to promote superficial understanding, rather than the true depth and rigour of knowledge that is a foundation for higher mathematics.

Within the materials the terms mastery and mastery with greater depth are used to acknowledge that all pupils require depth in their learning, but some pupils will go deeper still in their learning and understanding.

Mastery of the curriculum requires that all pupils:

• use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
• recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
• have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

4. Intelligent practice is a term used to describe practice exercises that integrate the development of fluency with the deepening of conceptual understanding. Attention is drawn to the mathematical structures and relationships to assist in the deepening of conceptual understanding, whilst at the same time developing fluency through practice.


7. This argument was advanced by the Advisory Committee for Mathematics Education on page 1 of its report: Raising the bar: developing able young mathematicians, December 2012.
A useful checklist for what to look out for when assessing a pupil’s understanding might be:

A pupil really understands a mathematical concept, idea or technique if he or she can:

• describe it in his or her own words;
• represent it in a variety of ways (e.g. using concrete materials, pictures and symbols – the CPA approach)8
• explain it to someone else;
• make up his or her own examples (and non-examples) of it;
• see connections between it and other facts or ideas;
• recognise it in new situations and contexts;
• make use of it in various ways, including in new situations.9

Developing mastery with greater depth is characterised by pupils’ ability to:

• solve problems of greater complexity (i.e. where the approach is not immediately obvious), demonstrating creativity and imagination;
• independently explore and investigate mathematical contexts and structures, communicate results clearly and systematically explain and generalise the mathematics.

The materials seek to exemplify what these two categories of mastery and mastery with greater depth might look like in terms of the type of tasks and activities pupils are able to tackle successfully. It should, however, be noted that the two categories are not intended to exemplify differentiation of activities/tasks. Teaching for mastery requires that all pupils are taught together and all access the same content as exemplified in the first column of questions, tasks and activities. The questions, tasks and activities exemplified in the second column might be used as deepening tasks for pupils who grasp concepts rapidly, but can also be used with the whole class where appropriate, giving all children the opportunity to think and reason more deeply.

National curriculum assessments

National assessment at the end of Key Stages 1 and 2 aims to assess pupils’ mastery of both the content of the curriculum and the depth of their understanding and application of mathematics. This is exemplified through the content and cognitive domains of the test frameworks.10 The content domain exemplifies the minimum content pupils are required to evidence in order to show mastery of the curriculum. The cognitive domain aims to measure the complexity of application and depth of pupils’ understanding. The questions, tasks and activities provided in these materials seek to reflect this requirement to master content in terms of both skills and depth of understanding.

Final remarks

These resources are intended to assist teachers in teaching and assessing for mastery of the curriculum. In particular they seek to exemplify what depth looks like in terms of the types of mathematical tasks pupils are able to successfully complete and how some pupils can achieve even greater depth. A key aim is to encourage teachers to keep the class working together, spend more time on teaching topics and provide opportunities for all pupils to develop the depth and rigour they need to make secure and sustained progress over time.

8. The Concrete-Pictorial-Abstract (CPA) approach, based on Bruner’s conception of the enactive, iconic and symbolic modes of representation, is a well-known instructional heuristic advocated by the Singapore Ministry of Education since the early 1980s. See https://www.ncetm.org.uk/resources/44565 (free registration required) for an introduction to this approach.
10. 2016 Key stage 1 and 2 Mathematics test frameworks, Standards and Assessments Agency
    www.gov.uk/government/collections/national-curriculum-assessments-test-frameworks
The structure of the materials

The materials consist of PDF documents for each year group from Y1 to Y6. Each document adopts the same framework as outlined below.

The examples provided in the materials are only indicative and are designed to provide an insight into:

- How mastery of the curriculum might be developed and assessed;
- How to teach the same curriculum content to the whole class, challenging the rapid graspers by supporting them to go deeper rather than accelerating some pupils into new content.

The assessment activities presented in both columns are suitable for use with the whole class. Pupils who successfully answer the questions in the left-hand column (Mastery) show evidence of sufficient depth of knowledge and understanding. This indicates that learning is likely to be sustainable over time. Pupils who are also successful with answering questions in the right-hand column (Mastery with Greater Depth) show evidence of greater depth of understanding and progress in learning.

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**Number and Place Value**

Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero

The Big Idea

Large numbers of six digits are named in a pattern of three: hundreds of thousands, tens of thousands, ones of thousands, mirroring hundreds, tens and ones. It is helpful to relate large numbers to real-world contexts, for example the number of people that a local sports arena can hold.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’ , ‘What happens if …?’ , and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<thead>
<tr>
<th>Number and Place Value</th>
<th>Number and Place Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mastery</strong></td>
<td><strong>Mastery with Greater Depth</strong></td>
</tr>
<tr>
<td>Explore 1 million:</td>
<td>Explore 1 million:</td>
</tr>
<tr>
<td>■ Write 1 million in digits.</td>
<td>■ How large would a stadium need to be to hold one million people?</td>
</tr>
<tr>
<td>■ Write down the number that is 1 more than 1 million.</td>
<td>■ How much would a million grains of rice weigh?</td>
</tr>
<tr>
<td>■ Write down the number that is 10 more than 1 million.</td>
<td>In June 2014 the population of the UK was approximately 64 100 000.</td>
</tr>
<tr>
<td>■ Write down the number that is 100 more than 1 million.</td>
<td>What is the current approximate population of the UK?</td>
</tr>
<tr>
<td>In June 2014 the population of the UK was approximately 64 100 000.</td>
<td>Is this number larger or smaller than 64 100 000?</td>
</tr>
<tr>
<td>Round this number to the nearest million.</td>
<td>How accurate is this figure in terms of the number of people in the UK at this moment?</td>
</tr>
</tbody>
</table>
### Number and Place Value

#### Selected National Curriculum Programme of Study Statements

Pupils should be taught to:
- read, write, order and compare numbers to at least 1,000,000 and determine the value of each digit
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers including through zero

#### The Big Idea

Large numbers of six digits are named in a pattern of three: hundreds of thousands, tens of thousands, ones of thousands, mirroring hundreds, tens and ones.
It is helpful to relate large numbers to real-world contexts, for example the number of people that a local sports arena can hold.

#### Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’, ‘What happens if …?’, and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<thead>
<tr>
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<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explore 1 million:</td>
<td></td>
</tr>
</tbody>
</table>
- Write 1 million in digits. 
- Write down the number that is 1 more than 1 million. 
- Write down the number that is 10 more than 1 million. 
- Write down the number that is 100 more than 1 million. | Explore 1 million: | 
- How large would a stadium need to be to hold one million people? 
- How much would a million grains of rice weigh? |

In June 2014 the population of the UK was approximately 64,100,000.
Round this number to the nearest million.

In June 2014 the population of the UK was approximately 64,100,000.
What is the current approximate population of the UK?
Is this number larger or smaller than 64,100,000?
How accurate is this figure in terms of the number of people in the UK at this moment?
Mastery

What can we say about 48 000?

It is [ ] less than 50 000.
It is made of 40 000 and [ ] together.
It is made of [ ] thousands.
It is made of [ ] hundreds.
It is made of [ ] tens.

Mastery with Greater Depth

Using all of the digits from 0 to 9, write down a 10-digit number.

What is the largest number you can write?
What is the smallest number you can write?

Write down the number that is one less than the largest number.
Write down the number that is one more than the smallest number.

Captain Conjecture says, ‘Using the digits 0 to 9 we can write any number, no matter how large or small.’
Do you agree?
Explain your reasoning.

The temperature at 6 a.m. was recorded each day for one week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Tues</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (°C)</td>
<td>1</td>
<td>−1</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>−2</td>
<td>−3</td>
</tr>
</tbody>
</table>

What was the coldest morning?
What was the warmest morning?
What is the difference in temperature between Monday and Tuesday?
Place the recorded temperatures in order from smallest to largest.

The temperature at 6 a.m. was recorded each day for one week.

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Tues</th>
<th>Wed</th>
<th>Thurs</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp (°C)</td>
<td>1</td>
<td>−1</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>−2</td>
<td>−3</td>
</tr>
</tbody>
</table>

What is the difference in temperature between the coldest day and the warmest day?
At what time of year do you think these temperatures were recorded?
Do you think it might have snowed during the week?
Explain your reasoning.
Addition and Subtraction

Selected National Curriculum Programme of Study Statements
Pupils should be taught to:
- add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers (e.g. $12462 - 2300 = 10162$)
- solve problems involving numbers up to three decimal places (Taken from Y5 Fractions, Decimals and Percentages)

The Big Ideas
Before starting any calculation is it helpful to think about whether or not you are confident that you can do it mentally. For example, $3689 + 4998$ may be done mentally, but $3689 + 4756$ may require paper and pencil.

Carrying out an equivalent calculation might be easier than carrying out the given calculation. For example $3682 - 2996$ is equivalent to $3686 - 3000$ (constant difference).

Mastery Check
Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’, ‘What happens if …?’, and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set out and solve these calculations using a column method.</td>
<td>True or False?</td>
</tr>
<tr>
<td>$3254 + \underline{} = 7999$</td>
<td>$3999 - 2999 = 4000 - 3000$</td>
</tr>
<tr>
<td>$2431 = \underline{} - 3456$</td>
<td>$3999 - 2999 = 3000 - 2000$</td>
</tr>
<tr>
<td>$6373 - \underline{} = 3581$</td>
<td>$2741 - 1263 = 2742 - 1264$</td>
</tr>
<tr>
<td>$6719 = \underline{} - 4562$</td>
<td>$2741 + 1263 = 2742 + 1264$</td>
</tr>
<tr>
<td></td>
<td>$2741 - 1263 = 2731 - 1253$</td>
</tr>
<tr>
<td></td>
<td>$2741 - 1263 = 2742 - 1252$</td>
</tr>
<tr>
<td>Explain your reasoning.</td>
<td>Using this number statement, $5222 - 3111 = 5223 - 3112$ write three more pairs of equivalent calculations.</td>
</tr>
<tr>
<td></td>
<td>Pupils should not calculate the answer to these questions but should look at the structure and relationships between the numbers.</td>
</tr>
</tbody>
</table>
### Mastery

Write four number facts that this bar diagram shows.

<table>
<thead>
<tr>
<th>9.5</th>
<th>3.8</th>
<th>5.7</th>
</tr>
</thead>
</table>

\[
\begin{align*}
\square + \square &= \square \\
\square + \square &= \square \\
\square - \square &= \square \\
\square - \square &= \square \\
\end{align*}
\]

### Mastery with Greater Depth

Use this number sentence to write down three more pairs of decimal numbers that sum to 3:

\[1.6 + 1.4 = 3\]

#### Captain Conjecture says, ‘When working with whole numbers, if you add two 2-digit numbers together the answer cannot be a 4-digit number.’

Do you agree?

Explain your reasoning.

#### Captain Conjecture says, ‘If you keep subtracting 3 from 397 you will eventually reach 0.’

Do you agree?

Explain your reasoning.
### Mastery

The table shows the cost of train tickets from different cities.

What is the total cost for a return journey to York for one adult and two children? How much more does it cost for two adults to make a single journey to Hull than to Leeds?

<table>
<thead>
<tr>
<th></th>
<th>York</th>
<th>Hull</th>
<th>Leeds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single</td>
<td>£13·50</td>
<td>£16·00</td>
<td>£11·00</td>
</tr>
<tr>
<td>Return</td>
<td>£24·50</td>
<td>£30·00</td>
<td>£20·00</td>
</tr>
<tr>
<td>Child</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single</td>
<td>£9·75</td>
<td>£11·00</td>
<td>£8·00</td>
</tr>
<tr>
<td>Return</td>
<td>£15·00</td>
<td>£18·50</td>
<td>£13·50</td>
</tr>
</tbody>
</table>

### Mastery with Greater Depth

Sam and Tom have £67·80 between them. If Sam has £6·20 more than Tom, how much does Tom have? *The bar model can help children solve these type of problems, please visit ncetm.org for further information on how to build understanding.*

\[
\begin{align*}
\text{Sam} & \quad + \, \text{£6·20} \\
\text{Tom} & \quad \text{£67·80} \\
\text{ё67·80} - \text{ё6·20} & = \text{ё61·60} \\
\text{ё61·60} \div 2 & = \text{ё30·80} \\
\text{Tom has £30·80}
\end{align*}
\]
Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- multiply numbers up to four digits by a 1 or 2-digit number using a formal written method, including long multiplication for 2-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to four digits by a 1-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared (²) and cubed (³)
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

The Big Ideas

Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn.

They recognise how to use their skills of multiplying and dividing in new problem solving situations.

Fractions and division are connected ideas: $36 \div 18 = \frac{36}{18} = 2; \frac{18}{36} = \frac{1}{2}$.

Factors and multiples are connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’ ‘What happens if …?’ and checking that pupils can use the procedures or skills to solve a variety of problems.
<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 is a multiple of 4 and a factor of 16</td>
<td>Captain Conjecture says, ‘Factors come in pairs so all numbers have an even number of factors.’</td>
</tr>
<tr>
<td>6 is a multiple of 3 and a factor of</td>
<td>Do you agree?</td>
</tr>
<tr>
<td>[ ] is a multiple of 5 and a factor of</td>
<td>Explain your reasoning.</td>
</tr>
<tr>
<td>[ ] is a multiple of [ ] and a factor of [ ]</td>
<td></td>
</tr>
<tr>
<td>[ ] is a multiple of [ ] and a factor of [ ]</td>
<td></td>
</tr>
<tr>
<td>A 50 cm length of wood is cut into 4 cm pieces.</td>
<td></td>
</tr>
<tr>
<td>How many 4 cm pieces are cut and how much wood is left over?</td>
<td></td>
</tr>
<tr>
<td>Fill in the blanks to represent the problem as division:</td>
<td></td>
</tr>
<tr>
<td>[ ] ÷ [ ] = [ ] remainder [ ]</td>
<td></td>
</tr>
<tr>
<td>Fill in the blanks to represent the problem as multiplication:</td>
<td></td>
</tr>
<tr>
<td>[ ] × [ ] + [ ] = 50</td>
<td></td>
</tr>
<tr>
<td>A 1 m piece of ribbon is cut into equal pieces and a piece measuring 4 cm remains.</td>
<td></td>
</tr>
<tr>
<td>What might the lengths of the equal parts be?</td>
<td></td>
</tr>
<tr>
<td>In how many different ways can the ribbon be cut into equal pieces?</td>
<td></td>
</tr>
</tbody>
</table>
### Mastery

Fill in the missing numbers in this multiplication pyramid.

```
   108
   6  3
  2
```

Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.

### Mastery with Greater Depth

Fill in the missing numbers:

\[
8 \div 2 = \boxed{} \div 4 = 32 \div \boxed{} = 64 \div \boxed{}
\]

Fill in the missing numbers:

\[
\boxed{} \div 120 = 117 \div 13 = 10800 \div \boxed{} = 234 \div \boxed{}
\]

Sally’s book is 92 pages long.

If she reads seven pages each day, how long will she take to finish her book?

A 5p coin has a thickness of 1·7 mm. Ahmed makes a tower of 5p coins worth 50p.

Write down the calculation you would use to find the height of the tower.
# Fractions

## Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (for example, \( \frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1 \frac{1}{5} \))
- add and subtract fractions with the same denominator and denominators that are multiples of the same number
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams
- recognise the per cent symbol (%) and understand that per cent relates to ‘number of parts per hundred’, and write percentages as a fraction with denominator 100, and as a decimal
- solve problems which require knowing percentage and decimal equivalents of \( \frac{1}{2} \), \( \frac{1}{4} \), \( \frac{1}{5} \), \( \frac{2}{5} \), \( \frac{4}{10} \) and those fractions with a denominator of a multiple of 10 or 25

## The Big Idea

Representations that may appear different sometimes have similar underlying ideas. For example \( \frac{1}{4} \), 0.25 and 25% are used in different contexts but are all connected to the same idea.

## Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’, ‘What happens if …?’; and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make each number sentence correct using (=), (&gt;) or (&lt;).</td>
<td>Write down two fractions where the denominator of one is a multiple of the denominator of the other.</td>
</tr>
<tr>
<td>(\frac{3}{4}) (\bigcirc) (\frac{1}{2})</td>
<td>Which is the larger fraction?</td>
</tr>
<tr>
<td>(\frac{3}{8}) (\bigcirc) (\frac{1}{2})</td>
<td>Explain your reasoning.</td>
</tr>
<tr>
<td>(\frac{3}{4}) (\bigcirc) (\frac{3}{8})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{4}) (\bigcirc) (\frac{2}{5})</td>
<td></td>
</tr>
<tr>
<td>(\frac{1}{2}) (\bigcirc) (\frac{1}{2})</td>
<td></td>
</tr>
<tr>
<td>(\frac{3}{4}) (\bigcirc) (\frac{3}{8})</td>
<td></td>
</tr>
<tr>
<td>(\frac{2}{5}) (\bigcirc) (\frac{4}{10})</td>
<td></td>
</tr>
<tr>
<td>(\frac{2}{5}) (\bigcirc) (\frac{5}{10})</td>
<td></td>
</tr>
<tr>
<td>Mastery</td>
<td>Mastery with Greater Depth</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>Mark and label on this number line where you estimate that $\frac{3}{4}$ and $\frac{3}{8}$ are positioned.</td>
<td>Russell says $\frac{3}{8} &gt; \frac{3}{4}$ because $8 &gt; 4$.</td>
</tr>
<tr>
<td>[Diagram of a number line with marks at 0, 1/2, 1]</td>
<td>Do you agree?</td>
</tr>
<tr>
<td></td>
<td>Explain your reasoning.</td>
</tr>
<tr>
<td>Choose numbers for each numerator to make this number sentence true.</td>
<td>Which is closer to 1?</td>
</tr>
<tr>
<td>15 &gt; [Blank] 10</td>
<td>$\frac{7}{8}$ or $\frac{23}{24}$</td>
</tr>
<tr>
<td></td>
<td>Explain how you know.</td>
</tr>
<tr>
<td>Chiz and Caroline each had two sandwiches of the same size. Chiz ate $\frac{1}{2}$ of his sandwiches. Caroline ate $\frac{5}{4}$ of her sandwiches. Draw diagrams to show how much Chiz and Caroline each ate. Who ate more? How much more?</td>
<td>Chiz and Caroline each had two sandwiches of the same size. Chiz ate $1 \frac{1}{4}$ of his sandwiches. Caroline ate $\frac{5}{4}$ of her sandwiches. Fred said Caroline ate more because 5 is the biggest number. Tammy said Chiz ate more because she ate a whole sandwich. Explain why Fred and Tammy are both wrong.</td>
</tr>
</tbody>
</table>
### Mastery

Each bar of toffee is the same. On Monday, Sam ate the amount of toffee shown shaded in A. On Tuesday, Sam ate the amount of toffee shown shaded in B.

How much more, as a fraction of a bar of toffee, did Sam eat on Tuesday?

![Diagram](image1.png)

Sam says he ate \(\frac{7}{8}\) of a bar of toffee.
Jo says Sam ate \(\frac{7}{16}\) of the toffee.

Explain why Sam and Jo are both correct.

Using the numbers 5 and 6 only once, make this sum have the smallest possible answer:

\[
\begin{array}{c}
\square + \square \\
15 & 10
\end{array}
\]

Using the numbers 3, 4, 5 and 6 only once, make this sum have the smallest possible answer:

\[
\begin{array}{c}
\square + \square \\
\square & \square
\end{array}
\]

Graham is serving pizzas at a party. Each person is given \(\frac{3}{4}\) of a pizza. Graham has six pizzas.

How many people can he serve? Draw on the pizzas to show your thinking.

![Diagram](image2.png)

Write your answer as a multiplication sentence.

### Mastery with Greater Depth

Each bar of toffee is the same. On Monday, Sam ate the amount of toffee shown shaded in A. On Tuesday, Sam ate the amount of toffee shown shaded in B.

![Diagram](image1.png)

Sam says he ate \(\frac{7}{8}\) of a bar of toffee.
Jo says Sam ate \(\frac{7}{16}\) of the toffee.

Explain why Sam and Jo are both correct.

Using the numbers 3, 4, 5 and 6 only once, make this sum have the smallest possible answer:

\[
\begin{array}{c}
\square + \square \\
\square & \square
\end{array}
\]

Graham is serving pizzas at a party. Each person is given \(\frac{3}{4}\) of a pizza.

Fill in the table below to show how many pizzas he must buy for each number of guests.

<table>
<thead>
<tr>
<th>Guests</th>
<th>Pizzas</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

When will he have pizza left over?
Krysia wanted to buy a coat that cost £80. She saw the coat on sale in one shop at \( \frac{1}{5} \) off. She saw the same coat on sale in another shop at 25% off.

Which shop has the coat at a cheaper price?
Explain your reasoning.

Express the yellow section of the grid in hundredths, tenths, as a decimal and as a percentage of the whole grid.
Do the same for the red section.
## Measurement

### Selected National Curriculum Programme of Study Statements

Pupils should be taught to:

- convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm²) and square metres (m²) and estimate the area of irregular shapes

### The Big Ideas

The relationship between area and perimeter is not a simple one. Increasing or decreasing area does not necessarily mean the perimeter increases or decreases respectively, or vice versa.

Area is measured in square units. For rectangles, measuring the length and breadth is a shortcut to finding out how many squares would fit into each of these dimensions.

### Mastery Check

Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as ‘Why?’, ‘What happens if …?’, and checking that pupils can use the procedures or skills to solve a variety of problems.

<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete this:</td>
<td>True or false?</td>
</tr>
<tr>
<td>$\frac{1}{2}$ kg = ____ g</td>
<td>1·5 kg + 600 g = 2·1 kg + 300 g</td>
</tr>
<tr>
<td>$\frac{1}{4}$ kg = _____ g</td>
<td>32 cm + 1·05 m = 150 cm – 0·13 m</td>
</tr>
<tr>
<td>Which has the greater mass?</td>
<td>$\frac{3}{4}$ l + 0.05 l = half of 1·6 l</td>
</tr>
<tr>
<td>$\frac{1}{5}$ kg or $\frac{1}{10}$ kg</td>
<td>Explain your reasoning.</td>
</tr>
</tbody>
</table>
### Mastery

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>The weight of a football is 400 g. How much do five footballs weigh in kilograms?</td>
<td>A football weighs 0.4 kg. Three footballs weigh the same as eight cricket balls. How many grams does a cricket ball weigh?</td>
</tr>
<tr>
<td>Joe and Kate are using two metre sticks to measure the height of the climbing frame. Their measurements are shown in the diagram.</td>
<td>A 1.2 m ribbon and a 90 cm ribbon are joined by overlapping the ends and gluing them together. The total length of ribbon needs to be 195 cm long. How much should the two pieces overlap?</td>
</tr>
</tbody>
</table>

### Mastery with Greater Depth

**Height of the Climbing Frame**

![Diagram of a climbing frame](image)

1 m

1 m

---

1 m

1 m
<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>A box weighs 1.3 kg. A box and two tins weigh 1.6 kg.</td>
<td>Here are some tins and boxes on two different scales.</td>
</tr>
<tr>
<td>How much does one tin weigh in grams?</td>
<td>How many grams does a tin weigh? How many grams does the box weigh?</td>
</tr>
</tbody>
</table>

Here are some tins and boxes on two different scales. How many grams does a tin weigh? How many grams does the box weigh?

Here is a picture of a square drawn on cm² paper.

Draw another rectangle with the same perimeter as this square. Do the two rectangles have the same area? Is this always, sometimes or never true of other pairs of rectangles with the same perimeter? Explain your reasoning.

How many other rectangles are there with the same perimeter as the square, where the sides are a whole number of cm? Show your workings.
<table>
<thead>
<tr>
<th>Mastery</th>
<th>Mastery with Greater Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamsa has some juice in a jug and he pours it into a different jug.</td>
<td>A litre of water is approximately a pint and three quarters.</td>
</tr>
<tr>
<td>Draw the level of the juice in the jug on the right.</td>
<td>How many pints are equivalent to 2 litres of water?</td>
</tr>
<tr>
<td><img src="image" alt="Juice levels" /></td>
<td>Using the approximation, when will the number of litres and the equivalent number of pints be whole numbers?</td>
</tr>
</tbody>
</table>
Geometry

Selected National Curriculum Programme of Study Statements
Pupils should be taught to:
- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- draw given angles, and measure them in degrees (°)
- identify:
  - angles at a point and one whole turn (total 360°)
  - angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180°)
  - other multiples of 90°
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles

The Big Ideas
During this year, pupils increase the range of 2-D and 3-D shapes that they are familiar with. With 3-D shapes they think about the faces as well as the number of vertices and through considering nets think about the 2-D shapes that define the 3-D shapes.

Pupils learn about a range of angle facts and use them to describe certain shapes and derive facts about them.

Regular shapes have to have all sides and all angles the same. Although non-square rectangles have four equal angles, the fact that they do not have four equal sides means that they are not regular.

Some properties of shapes are dependent upon other properties. For example, a rectangle has opposite sides equal because it has four right angles. A rectangle is defined as a quadrilateral with four right angles. It does not have to be defined as a quadrilateral with four right angles and two pairs of equal sides.

Mastery Check
Please note that the following columns provide indicative examples of the sorts of tasks and questions that provide evidence for mastery and mastery with greater depth of the selected programme of study statements. Pupils may be able to carry out certain procedures and answer questions like the ones outlined, but the teacher will need to check that pupils really understand the idea by asking questions such as 'Why?,' 'What happens if …?', and checking that pupils can use the procedures or skills to solve a variety of problems.
### Mastery

The circle is divided into quarters by the two diameter lines and four angles A, B, C and D are marked.

Are the statements below true or false?
- Angle C is the smallest angle.
- Angle D is the largest angle.
- All the angles are the same size.
- Angle B is a right angle.
- Angle B is an obtuse angle.

Explain your reasoning.

![Circle with angles](image)

### Mastery with Greater Depth

In the questions, below all of Harry’s movement is in a clockwise direction.

If Harry is facing North and turns through 180 degrees, in which direction will he be facing?

If Harry is facing South and turns through 180 degrees, in which direction will he be facing?

What do you notice?

If Harry is facing North and wants to face SW how many degrees must he turn? From this position how many degrees must he travel through to face North again?

![Compass](image)

### Identify the regular and irregular quadrilaterals.

Identify the regular and irregular quadrilaterals.

![Quadrilaterals](image)

*Pupils should recognise that a square is the only regular quadrilateral and there are two within this set.*

### Which of these statements are correct?

- A square is a rectangle.
- A rectangle is a square.
- A rectangle is a parallelogram.
- A rhombus is a parallelogram.

Explain your reasoning.
### Mastery

What shapes do you make when these 2-D representations (nets) are cut out and folded up to make 3-D shapes?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td><img src="image1.png" alt="Diagram A" /></td>
</tr>
<tr>
<td>B</td>
<td><img src="image2.png" alt="Diagram B" /></td>
</tr>
</tbody>
</table>

### Mastery with Greater Depth

Draw the 2-D representation (net) that will make this cuboid when cut out and folded up.

![Diagram C](image3.png)
### Statistics

**Selected National Curriculum Programme of Study Statements**

Pupils should be taught to:

- solve comparison, sum and difference problems using information presented in a line graph
- complete, read and interpret information in tables, including timetables

**The Big Ideas**

Different representations highlight different aspects of data.

It is important to be able to answer questions about data using inference and deduction, not just direct retrieval.

**Mastery Check**

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<table>
<thead>
<tr>
<th>Highway Rd</th>
<th>Rain Rd</th>
<th>Coldcot Rd</th>
<th>Westland Rd</th>
<th>Bod Rd</th>
<th>Kingswell Rd</th>
<th>Long Rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>06:50</td>
<td>07:00</td>
<td>07:11</td>
<td>07:18</td>
<td>07:29</td>
<td>07:33</td>
<td>07:45</td>
</tr>
<tr>
<td>07:25</td>
<td>07:25</td>
<td>07:41</td>
<td>07:59</td>
<td>08:09</td>
<td>08:15</td>
<td>08:30</td>
</tr>
<tr>
<td>08:45</td>
<td>08:55</td>
<td>09:04</td>
<td>09:11</td>
<td>09:16</td>
<td>08:14</td>
<td>08:30</td>
</tr>
<tr>
<td>09:10</td>
<td>09:19</td>
<td>09:28</td>
<td>09:38</td>
<td>09:47</td>
<td>09:53</td>
<td>10:05</td>
</tr>
<tr>
<td></td>
<td>09:53</td>
<td>10:02</td>
<td>10:11</td>
<td>10:16</td>
<td>10:21</td>
<td></td>
</tr>
</tbody>
</table>

Use the bus timetable to answer the following questions:

- On the 6:50 bus how long does it take to get from Highway Rd to Westland Rd?
- Can you travel to Long Rd on the 8:45 bus?
- Which journey between Rain Rd and Kingswell Rd takes the longest time, the bus that leaves Rain Rd at 7:25 or the bus that leaves Rain Rd at 7:41?
- Explain your reasoning.

<table>
<thead>
<tr>
<th>Highway Rd</th>
<th>Rain Rd</th>
<th>Coldcot Rd</th>
<th>Westland Rd</th>
<th>Bod Rd</th>
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<td>09:53</td>
<td>10:05</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the bus timetable to answer the following questions:

- If you needed to travel from Coldcot Rd and arrive at Kingswell Rd by 8:20, which would be the best bus to catch?
- Explain why.
- Which journey takes the longest time?
### Mastery

Use the line graph to answer the following questions:
Approximately how much does the average child grow between the ages of 1 and 2?
Do they grow more between the ages of 1 and 2 or 7 and 8?

#### The growth of children between the ages of 1 and 8

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
</tr>
<tr>
<td>6</td>
<td>120</td>
</tr>
<tr>
<td>7</td>
<td>130</td>
</tr>
<tr>
<td>8</td>
<td>140</td>
</tr>
</tbody>
</table>

### Mastery with Greater Depth

Use the line graph to answer the following questions:
From the graph can you predict the approximate height of an average 10 year old? Explain how.
Consider what might be the similarities and differences between this graph and a graph of the average height of teenagers.

#### The growth of children between the ages of 1 and 8

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
</tr>
<tr>
<td>3</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
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<tr>
<td>6</td>
<td>120</td>
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</tr>
<tr>
<td>8</td>
<td>140</td>
</tr>
</tbody>
</table>