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Learning GCSE mathematics through discussion: what are the effects on students?

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Currently, the teaching of retake GCSE mathematics in further education is predominantly teacher-centred and transmission-oriented. This paper argues that this approach is ineffective for students' learning and for their attitudes towards learning and that a student-centred, collaborative approach to learning, where discussion and reflection are central, can prove more effective in developing students' understanding of and attitudes towards mathematics. A teaching resource encouraging the discussion-based learning of algebra was developed and introduced to teachers of retake GCSE classes from 44 further education colleges. Outcomes relating to students' learning and attitudes towards learning were evaluated. The results indicate that learning increased with both the number of activities used and the degree to which the teaching was reported (by students) as student-centred. Learning gains were modest, reflecting the difficulty of algebraic concepts for these students. Students' confidence, motivation and anxiety remained largely unchanged, in contrast to a control group, where the more transmission-based approaches were associated with a small decline in these aspects.

Introduction

The quality of GCSE mathematics teaching in further education (FE) colleges has been severely criticized for a long time. Each year approximately half of all students entered for GCSE fail to attain grade C or above and substantial numbers elect to repeat the course over one year. Inspection evidence suggests that colleges frequently over-recruit and that many of the students withdraw before completing courses. Class sizes are small and the average rate of attendance is about two-thirds (Further Education Funding Council [FEFC], 1997, 1999). Although with such low numbers one might expect teachers to accommodate individual learning needs, approaches to teaching rarely do this. Most teaching may be characterized as transmission oriented; the the whole syllabus is 'delivered' again over one year at double pace. Students' responses are mostly passive and many become demotivated

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(FEFC, 1997, para. 19 and 27). Throughout this process teachers appear more concerned that students acquire procedural fluency rather than conceptual understanding or problem-solving strategies (FEFC, 1999, para. 57). Students tend to work as individuals and there is little opportunity for the collaborative discussion of ideas and approaches. Unsurprisingly perhaps, this approach is reported as ineffective in terms of learning and attitudes towards learning. Many of the students who retake GCSE mathematics fail to improve their GCSE grade (FEFC, 1999, para. 63) and for many the experience 'reinforces failure and decreases motivation' (FEFC, 1997, para. 38). This is supported by the recent government white paper that reports that only around 20% of students gain a higher grade at GCSE than they did the first time around (Kelly, 2005).

This paper reports on a design research study that seeks to transform such practices. Design research is an emerging paradigm for the study of learning through the systematic design of teaching strategies and tools. The beginnings of this movement are usually attributed to Brown (1992) and Collins (1992), though, in a sense, the concept of design research was an approach simply waiting to be named (Schoenfeld, 2004). There have recently been several attempts to develop theoretical bases for design-based research (see, for example, Cobb *et al.*, 2003; Design-Based Research Collective (DBRC), 2003; Schoenfeld, 2004). These typically elicit the following characteristics: it is highly interventionist; the twin goals of designing learning environments and developing learning theories are intertwined; development and research take place through an iterative process of design, enactment, analysis and redesign; its pragmatic focus results in theories and resources that communicate to practitioners and other educational designers.

The research reported here is the result of the second iteration in a design research process that seeks to develop approaches and materials that are effective in typical FE contexts. It is based on earlier research conducted by Swan (2000). This study involved the design of discussion-based collaborative teaching resources in five mathematical topics based on design principles drawn from diagnostic teaching research (Bell *et al.*, 1985; Bell, 1993). It involved four teachers over 2 years. During the first year, the teachers adopted their usual practices. During the second year they used the new activities. The resources facilitated significant learning gains only when they were used in student-centred ways, i.e. when students were allowed to discuss and reformulate their own ideas. The teachers that used the resources in more teacher-centred ways, where they continued to view their task as transmitting facts and skills, were much less successful.

Building on this research, the teaching resource *Learning mathematics through discussion and reflection: algebra at GCSE* (Swan & Green, 2002) was designed, developed and circulated to all colleges. This paper reports the effects of using this resource with teachers and students from 44 FE colleges.

This research has further relevance in the light of the recent work by the Department for Education and Skills (DfES) Standards Unit Mathematics Team. This has now reached fruition and a collection of resources based on similar design principles has recently been sent to all colleges, schools, prisons and work-based learning providers (Swan & Wall, 2005). Thus the research reported here offers some hard evidence that these resources are effective in typical FE contexts, if used in student-centred ways.

The design of the teaching resource

The algebra teaching resources comprise classroom materials, teaching guidelines, illustrative video clips of the resources being used in three classrooms and questions for reflection and discussion to enable teachers to think more deeply about the issues involved. These resources were presented on a CD-ROM, with an accompanying introductory 20-minute video. Altogether, the teaching resources supported 10 'lessons' that would occupy approximately 17–20 hours of class time.

The context of algebra was chosen for three reasons. Firstly, algebraic thinking is fundamental to all mathematical work. Algebra is the language of generalization that describes the underlying structures of mathematics. One characteristic of low-attaining students is that they do not appear to appreciate such structures and attempt to memorize discrete results rather than comprehend connected ideas. Thus they learn that "speed=distance÷time" and "time=distance÷speed" as isolated fragments of knowledge, rather than alternative descriptions of the same mathematical relationship. Secondly, algebra was one topic that did not show much improvement during the first iteration of the research. Students had shown little understanding of algebraic concepts and we wanted to see if this situation might be improved if the intensity of the reflection and discussion was increased.

The theoretical principles underpinning the design of the resources are described fully in Swan (2005). They may be summarized as follows:

- lessons are conducted in supportive social contexts;
- lessons consist of rich, challenging tasks;
- students are encouraged to make mistakes and learn from them;
- teaching emphasizes methods and reasons rather than just answers;
- students create links between mathematical topics;
- the purpose of each lesson is communicated clearly to students;
- appropriate use is made of technology.

The intention was to show teachers generic approaches for fostering collaborative cultures within their classes. Through these, algebraic concepts and their representations could be identified, described and discussed intensively and alternative conceptions (or misconceptions) explicitly recognized and worked on. Students could be encouraged to be creative, make decisions, explain, prove, reflect and interpret. There were multiple entry points to most tasks, allowing students to take on challenges at different levels. Cognitive conflict was generated through the careful choice and juxtaposition of examples. Links were drawn through the use of multiple representations of the same idea. We thus sought to communicate algebra as active construction rather than an inert body of facts and skills.

The materials were designed around three generic activity types.

Evaluating the validity of statements and generalizations

These activities were intended to encourage students to reflect on common convictions concerning mathematical concepts. A number of generalizations were provided and students were asked to examine each one, decide upon its validity, then justify this decision with examples, counterexamples and explanations. Many statements contained common mistakes or misconceptions. The focus of attention was thus placed on reasoning. Statements were expressed in words or symbols or both.

Collaborative discussion and group accountability was encouraged by asking each group to make a poster of their results. This involved pasting down each statement under one of the headings 'always true, sometimes true or never true' and then surrounding the statement with justification and explanation.

This type of activity was used to encourage reflection on generalizations concerning the laws of arithmetic, the meaning of letters in algebra and the difference between identities, equations and inequations. Some examples are shown in Figure 1.

Interpreting and classifying multiple representations of mathematical objects

Mathematical concepts have many representations, from the conventionally agreed to the less formal. In algebra these include words, diagrams, symbolic formulas, tables and graphs. Card sorting activities allowed these representations to be shared, interpreted, compared and classified. In noticing the 'sameness' or 'difference' in representations students began to create and refine concepts and definitions.

Again, students were encouraged to record their groupings of cards using posters annotated with reasoning. Quite large card sets were used. Figure 2 shows part of one set. In this example students were first asked to match the corresponding words and symbols (not all are shown). This focused attention on the order of operations. Students were then asked to match these pairs to tables of data and area diagrams. As each area may be found in different ways, it was hoped that this might provide support for justifying why different expressions are equivalent. Where cards were missing, then students had to create these for themselves. This activity provoked much discussion of common algebraic mistakes, such as $2(n+6)=2n+6$, $3n^2=(3n)^2$ and so on.

$12a>12$	$12-a<12$	$\sqrt{a}<a$	$a^2>a$
If you multiply 12 by a number, the answer will be greater than 12.	If you take a number away from 12, your answer will be less than 12.	The square root of a number is less than the number.	The square of a number is greater than the number

Figure 1. Some typical generalizations to classify: always, sometimes or never true?

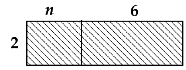
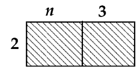
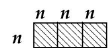
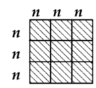
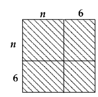
<table>
<tr><td>$\frac{n+6}{2}$</td><td>$3n^2$</td><td>Multiply n by two, then add six.</td><td>Square n, then multiply by three</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td></td><td>81</td><td>100</td></tr></table></td><td></td></tr>
<tr><td>$9n^2$</td><td>$2n+6$</td><td>Add six to n then multiply by two.</td><td>Multiply n by three, then square the answer</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td>14</td><td>16</td><td>18</td><td>20</td></tr></table></td><td></td></tr>
<tr><td>$2(n+3)$</td><td>$\frac{n}{2}+6$</td><td>Add three to n then multiply by two.</td><td>Add six to n then divide by two</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td>10</td><td>12</td><td>14</td></tr></table></td><td></td></tr>
<tr><td>$(3n)^2$</td><td>$2(n+6)$</td><td>Multiply n by two then add twelve</td><td>Divide n by two then add three</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td>3</td><td></td><td>27</td><td>48</td></tr></table></td><td></td></tr>
<tr><td>$(n+6)^2$</td><td>$2n+12$</td><td>Square n, then multiply by nine</td><td>Add six to n then square the answer</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td></td><td>81</td><td>144</td></tr></table></td><td></td></tr>
</table>

Figure 2. Multiple representations: a sample of the cards used for sorting

Creating and solving new problems

Students often see mathematics as something that is 'done to them' rather than as an opportunity for creative endeavour. In the teaching resources we sought to remedy this by asking students to create their own problems and examples. Other students were then invited to solve them. The originators and the solvers worked together to see where difficulties had emerged. Such tasks engaged students in the structure of problems and made explicit the processes of doing and undoing which permeate mathematics. One student carried out an operation (e.g. created an equation, expanded an expression), the other then reversed that operation (solved the equation, factorized the expression). The doer assisted the undoer when he or she became stuck. Thus they learned to resolve difficulties collaboratively.

Students would frequently create problems more difficult than the teacher would have chosen to give them. For example, six students in one class generated the equations in Figure 3. They each began with a known value for x and generated their equations step-by-step. Students found creating equations much easier than solving them because they could choose to use any operation they desired at each step. As they now 'owned' the procedures that created the equations, they were in a good position to advise other students when they became stuck while trying to solve them.

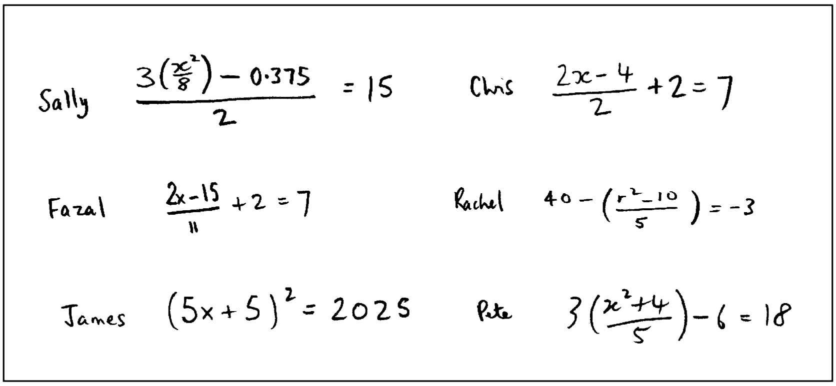


Figure 3. Examples of equations created by GCSE retake students in one class

The teacher sample

Invitations to join the second stage of the project were sent to all FE and sixth form colleges in England. Seventy colleges applied to participate and the first 44 applications were accepted. One participant from each college was invited to attend three workshops; a residential two-day introduction, followed by two one-day follow-up workshops. Other teachers in the same colleges were invited to act as a 'control' group, returning information on their normal ways of working. This was to enable us to be able to compare the discussion-based lessons with conventional approaches to teaching algebra in these colleges.

In total, 28 teachers attended all the workshops and returned student data (Table 1). A further 20 teachers from the same colleges returned student data without attending the workshops. Teachers were categorized into three groups according to how many of the discussion-based lessons they used and their involvement in the research. Seventeen teachers used 'many' of the lessons. This means that they used at least seven of the (notionally one hour) lessons from the supplied resources. A further 17 teachers used a 'few' lessons, i.e. they used from three to six lessons. In total, the 'many' group taught exactly twice as many lessons as the 'few' group. It should be emphasized that the teachers who taught 'none' of the discussion lessons taught a similar amount of algebra, but through 'chalk and talk' coupled with practice on textbook- and worksheet-based exercises.

Table 1. Teachers' participation in the research

Group	No. of discussion lessons taught	No. who attended workshops	No. who participated 'at a distance'	Total
'Many'	7–14	14	3	17
'Few'	3–6	14	3	17
'None'	0	0	14	14
Totals		28	20	48

The student sample

Teachers returned questionnaires for 834 students at the beginning of the year. Most students began the course with grades D (64%) or E (20%) in GCSE mathematics. By the time of the post-course questionnaires the total number of students who had attended at least 60% of lessons and who completed questionnaires a second time was 334. The reasons for the attrition in the sample were: students' poor attendance in lessons, absence during the administration of one or other of the questionnaires, students withdrawing from the GCSE course or teachers not administering the post-test questionnaires.

The final sample of 16–21-year-old students was almost identical in nature to the initial sample of 834 in terms of prior mathematical attainment and curriculum experience. We believe that the final reduced sample was drawn from a sufficiently diverse range of colleges to be representative of the general student population on GCSE retake courses.

The variation in teachers' practices

Of course, teachers used the resources in a variety of ways. This was monitored by asking both teachers and students to report on their methods. For example, students were given a list of 14 practices (see Table 2) and were asked to rate the frequency with which the teacher used each one on a five-point scale ranging from 5, 'almost always', to 1, 'almost never'. These statements were categorized as teacher-centred (T) or student-centred (S).

'Teacher-centred' describes practices that one would expect to arise from a transmission-oriented belief system. The teacher directs the work, predigests and organizes the material, gives clearly prescribed instructions, teaches everyone at once in a predetermined manner and emphasizes practice for fluency over discussion for meaning. There is little room for creativity. Above all, teaching is seen as the transmission of definitions and methods to be practised.

'Student-centred' describes practices arising from a constructivist position. This approach implies that the teacher takes students' knowledge and mistakes into account when deciding what to teach, treats students as individuals rather than a homogeneous block, is selective and flexible about what is covered and allows students to make decisions, compare different approaches and create their own methods. Mathematics is seen as a subject open for discussion.

It was found helpful to devise a scale of 'teacher centredness' using these statements. This was done by reverse scoring the ratings for the student-centred items and then adding them to the ratings for the teacher-centred items. Thus higher scores represent more teacher-centred behaviours. [The total score from the 14 items ranges from 14 to 70. It proved to have a Cronbach reliability coefficient $\alpha=0.73$ ($n=360$).]

Table 2 shows that little difference was reported by students between the teaching styles of teachers that taught 'none' or 'few' lessons, but they reported that teachers who used at least 7 hours of the discussion material were significantly more

Table 2. Student post-test views of teachers' practices

No.		The teacher …	None (*n*=97)	Few (*n*=130)	Many (*n*=135)
3	T	shows us which method to use, then asks us to use it.	4.0	4.0	3.7
12	T	tells us which questions to do.	4.0	4.1	3.4
1	T	asks us to work through practice exercises.	3.8	3.7	3.6
14	T	teaches algebra separately from other topics.	3.6	3.2	3.0
8	T	expects us to follow the textbook or worksheet closely.	3.4	3.6	3.2
15	S	encourages us to make and discuss mistakes.	3.3	3.2	3.6
2	T	expects us to work mostly on our own, asking a neighbour from time to time.	3.2	3.0	2.7
5	S	asks us to compare different methods for doing questions.	3.0	3.0	3.4
6	S	shows us how algebra links to other topics. (Like number or geometry)	3.0	3.1	3.1
9	S	expects us to learn through discussing our ideas.	3.0	3.1	3.6
13	T	shows us just one way of doing each question	2.6	2.7	2.4
11	S	lets us invent and use our own methods.	2.5	2.3	2.9
10	S	asks us to work in pairs or small groups	2.4	2.7	3.1
4	S	lets us choose which questions we do.	2.1	1.9	2.5

The first column shows the order of the items in the questionnaire.
The second column indicates whether the statement was regarded as teacher-centred or student-centred for scaling.
Columns 4–6 indicate the mean ratings given to each statement. Ratings were: 1=almost never; 2=occasionally; 3=half the time; 4=most of the time; 5=almost always.
The table is rank ordered with the most common activity for the teachers who used normal approaches ('none') coming first.

student-centred in their approach. In classes where 'many' activities were used, each of the seven teacher-centred statements was given a lower mean rating than in the classes where 'none' or 'few' activities were used. Similarly, each of the seven student-centred statements was given a higher mean rating than in the classes where 'none' or 'few' activities were used. In the 'many' classes there was a much greater emphasis on learning through discussion, particularly through the discussion of mistakes. This should have occurred through the card matching activities. Students also recognized that their teachers were encouraging the consideration of alternative methods. Again, this was encouraged within the teaching activities.

Notice, however, that even in the 'many' classes the most common teacher activity was to show students a method and then ask them to use it. The teaching activities designed for this project were not intended to remove the need for direct instruction. For example, in the lesson on creating and solving equations the teacher begins by demonstrating a method for creating an equation. Students are expected to use this method to design their own. Thus 'following a method' may still allow space for creativity. Practice exercises also remained commonplace, as one would expect. In general, therefore, the evidence seems to suggest that the more the discussion-based resources were used, the more student-centred the teaching became.

Students' learning

An algebra test was designed to assess the following range of elementary algebra concepts and skills:

- evaluating expressions involving numbers;
- simplifying simple algebraic expressions;
- substituting numbers into formulas;
- interpreting an expression set in a simple everyday context;
- extending a linear sequence and finding the nth term;
- constructing an algebraic expression from a simple everyday context;
- constructing an equation from a simple everyday context;
- solving linear equations;
- handling simple inequalities;
- rearranging formulas.

Several assessment tasks were constructed under each of the above headings, and each was scored on a two- or three-point scale. Full marks were given for a completely correct answer and part marks were obtained if the student showed some understanding. The resulting test comprised 29 items and was scored out of a total of 60 marks. Figure 4 illustrates some items that were used on the algebra test, while Table 3 shows some of the results on these items.

In total, 782 of the sample of students took the algebra pre-test, but only 312 of these were 16–21 years old, attempted both the algebra pre- and post-course questionnaires and also attended at least 60% of their mathematics lessons. Although this is a dramatic reduction from the original sample (but familiar in this context), the reduced sample is representative of the original one in terms of previous mathematics attainment.

It was informative to evaluate the relationship between the algebra improvement and both the number of and the manner in which lessons were taught. To this end, the results from the 'none', 'few' and 'many' categories were divided into two approximately equal subgroups according to whether those students were taught in a predominantly student-centred or teacher-centred manner. Students' mean ratings for each teacher are an unbiased guide to this. Individual teachers in each category

Substitution

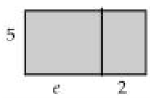
3a)	If $y = 1 + 4x$ and $x = 3$, then $y =$ ....
3b)	If $A = 3r^2$ and $r = 4$, then $A =$ ....
3c)	If $A = \frac{h(a+b)}{2}$ and $a = 4$, $b = 10$ and $h = 7$, then $A =$ ......

Constructing expressions

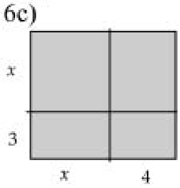
6a) A plumber charges £30 to come to your house plus an extra £20 for each hour that the job takes. A job takes x hours. How much does the plumber charge?

Write down an algebra expression for the area of each shape:

6b)



6c)



Constructing equations

A piece of rope 60 metres long is cut into two pieces. One piece is x metres long and the other is y metres long. Write down two equations. Each equation should use x, y and 60.

7a) .... = 60

7b) x = ...

There are 60 minutes in one hour. There are x minutes in y hours. Write down two equations. Each equation should use x, y and 60.

7c) x = ...

7d) y = ...

Solving equations

8a)	$2x + 7 = 45$
8b)	$2x + 12 = 5x$
8c)	$\frac{2x-1}{3} = 5$

Figure 4. Some items that were used on the algebra test

were first rank ordered according to the mean teacher-centredness scale ratings given by their students and then each group was split at the median rating so that samples of roughly equal size were obtained.

A two-way MANOVA was conducted on these and the results are given in Table 4. The mean changes show that within each group a student-centred approach resulted in a greater improvement in performance than a more teacher-centred approach. The greatest mean improvement (9 marks, 15%) was therefore observed with the 59 students who were taught many of the activities in student-centred ways. The results show statistically significant gains ($P<0.001$) from pre- to post-course algebra results overall, with significant interactions between the number of discussion lessons used from pre- to post-course algebra results ($P<0.001$) and for the student- or teacher-centredness from pre- to post-course algebra results

Table 3. Performances on selected algebra items by number of activities used

Question	None		Few		Many	
	Pre	Post	Pre	Post	Pre	Post
3a	59	66	54	61	44	65
3b	26	35	35	45	26	40
3c	48	48	44	45	35	55
6a	41	46	43	47	28	49
6b	9	5	11	24	7	31
6c	6	8	12	21	4	23
7a	45	42	53	54	34	50
7b	22	20	29	33	19	31
7c	12	7	18	18	10	15
7d	4	4	4	7	4	4
8a	49	60	46	66	41	65
8b	22	28	18	34	15	30
8c	12	25	27	23	17	33

($P<0.001$). Thus it does appear that the greater the number of discussion lessons used and the more student-centredness the teaching style, then the greater the gains in algebra learning. These gains are not very large, however, and this reflects the difficulty of the test for these students.

Students' attitudes

When students are confronted by high stakes examinations, such as the GCSE, teachers tend to adopt transmission styles and this lowers the self-esteem of students that prefer more active and creative methods (Harlen & Deakin Crick, 2002). It was hoped that these materials would help to reverse this process. In order to consider the effects of the programme on the confidence, anxiety and motivation of students we used three attitudes scales: the Confidence in Learning Mathematics Scale, the Effectance Motivation Scale and an adaptation of the Mathematics Anxiety Scale (Fennema & Sherman, 1976), in which the word 'mathematics' was replaced by 'algebra' throughout. These measure the degree to which students feel able to accept mathematical challenge, the degree to which they feel mathematics is a rewarding experience and the degree to which they feel anxiety when tackling algebra questions.

The results were somewhat surprising. The classes who had not used any of the discussion activities showed a decline ($P<0.05$) in their overall confidence in their ability to do mathematics, in their motivation towards the subject and their anxiety about algebra increased. In contrast, there were no statistically significant changes in the confidence and motivation of students in the classes that used the discussion activities.

Table 4. Algebra scores, by student view of teacher style and number of activities used

No. of lessons	Student evaluation of teacher-centredness	Mean pre-test (max 60)	Mean post-test (max 60)	Change
None	Student-centred (n=43)	18.02	22.26	+4.23
	Teacher-centred (n=42)	17.64	17.52	−0.12
Few	Student-centred (n=59)	18.63	23.08	+4.46
	Teacher-centred (n=55)	19.22	22.58	+3.36
Many	Student-centred (n=59)	14.12	23.10	+8.98
	Teacher-centred (n=54)	17.20	22.13	+4.93
Total	Student-centred (n=161)	16.81	22.87	+6.06
	Teacher-centred (n=151)	18.06	21.01	+2.95
	No. of lessons			
Total	None (n=85)	17.84	19.92	+2.08
	Few (n=114)	18.91	22.84	+3.93
	Many (n=113)	15.59	22.64	+7.05
Totals	Mean $\pm$ SD (n=312)	17.42 $\pm$ 9.11	21.97 $\pm$ 10.36	+4.55

MANOVA

Effect	Df	F	Significance
Teacher-centredness	1	0.2	n.s.
No. of lessons	2	1.6	n.s.
Teacher-centredness × No. of lessons	2	1.0	n.s.
Time (pre to post)	1	98.1	$P<0.001$
Teacher-centredness × Time	1	13.3	$P<0.001$
No. of lessons × Time	2	10.6	$P<0.001$
Teacher-centredness × No. of lessons × Time	2	1.5	n.s.

n.s., not significant.

Conclusions

The study reported here confirms much of what we already know about GCSE retake courses. Existing teaching methods are mostly transmission-oriented and unsuccessful in terms of algebra learning, with indications to suggest that students lose confidence, become less motivated and show increased anxiety about the subject.

The interventions decribed here, however, offer a radical departure to the conventional approaches to teaching these students. The resources encourage and support the implementation of collaborative, discussion-based approaches to learning algebra. As these are used by teachers, even for the first time, there is evidence here to suggest that learning is enhanced, particularly when they are used in student-centred ways. In particular, this means that students' existing knowledge and misunderstandings are brought to the surface and discussed in the lessons. The greatest gains (approximately one standard deviation) were made in the group that used many lessons in student-centred ways. The more student-centred approaches seem to have prevented a general decline in confidence and motivation that may occur when traditional didactic teaching approaches are used in FE classrooms.

These results are encouraging when seen in the light of the current DfES resources produced for the Success for All initiative, which are being widely disseminated in autumn 2005. They should go some way towards answering the perennial teachers' question: 'How do you know it will work?'.

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