Mathematics Matters

Final Report

National Centre
for Excellence in the
Teaching of Mathematics

www.ncetm.org.uk

Acknowledgement

The National Centre for Excellence in the Teaching of Mathematics (NCETM) is greatly indebted to Dr. Malcolm Swan, University of Nottingham, who chaired the project, devised the activities and co-wrote this report.

The direct involvement and leadership of Peter Lacey and Sarah Mann (ECARDA Ltd.), in project management, organisation of events and collection and analysis of data, also contributed greatly to the completion of the final report.

We wish to express our gratitude to the many mathematics teachers and other stakeholders, listed in Appendix 2, who contributed their time, views, and experiences. Our thanks also go to the NCETM advisory group and others who contributed to the critical review of the report.

CONTENTS

The following additional information is available on the NCETM portal at **www.ncetm.org.uk**. This may be of interest to those who would like to learn more about the background to the study, the methodology employed and the tools provided to and by participants.

- **Stimulus quotations**

Some extracts from writing over the last 100 years that describe characteristics of learning and teaching mathematics. (Sent out with the invitation to the initial conference in order to set an historical context.)

- **Lesson accounts**

Fifty-seven lesson accounts received at the colloquia, all of which have been checked and validated.

- **Discussion paper presented at the introductory conference**

What constitutes the effective learning of mathematics?
A discussion paper for the NCETM by Malcolm Swan

- **A summary of the report**

EXECUTIVE SUMMARY

Background

Over the past year, the National Centre for Excellence in the Teaching of Mathematics (NCETM) has undertaken a consultation, Mathematics Matters, to review and describe the values and practices considered to be most important and effective by the mathematics education community. The purpose of the consultation is:

- to establish an agreed set of valued learning outcomes and an agreed set of principles, grounded in evidence, that underpins the learning and teaching of mathematics
- to produce a collection of lesson accounts from a range of different settings in order to illustrate what the values and principles might look like in the classroom
- to ascertain the relative values put onto the different learning outcomes in an ideal case and to compare these with perceptions of relative values in practice
- to identify any barriers to the values and principles being translated into practice, and to propose strategies for overcoming them.

A draft set of values and principles for teaching and learning were established at an initial conference in London, in May 2007. These were then discussed, amended and exemplified through lesson accounts at a series of regional colloquia over the following year. A first progress report was issued by the NCETM immediately after the initial conference and a second progress report was issued in December 2007. This is the final report on the first two phases. Full versions of all the reports, the stimulus materials used, and a database of responses and lesson accounts may be found on the NCETM portal[1]. To date, we have consulted with over 150 mathematics educators, with representation from early years and primary teachers, secondary teachers, adult education teachers and teacher educators.

Participants have echoed Cockcroft (1982, para 242) in recognising that it is not possible to define a single 'best practice' in mathematics teaching. There are many different types of learning, and a wide range of teaching methods will need to be deployed, appropriate to the learners and the particular learning outcomes desired. Mathematics Matters attempted to clarify the types of learning that the mathematics community values most highly and identify underlying principles that might be helpful in guiding the choice of appropriate teaching methods. It also sought to identify common reasons why teachers do not always choose appropriate methods.

While recognising that this is only the beginning of a national consultation, the findings suggest that there are surprisingly high levels of agreement with the values and principles that are set out below. Much of what follows may sound like 'peace, motherhood and apple pie', but its implications are far-reaching. Also, while noting that the views expressed in this report are those of the participants alone, the NCETM fully endorses the values and principles agreed. The NCETM therefore looks forward to this report stimulating further debate and providing a basis for a wider consensus among the community of mathematics educators, assessment designers and policy makers in this country.

Outcomes

1. Valuing what is important

Participants reiterated the central importance of mathematics in the curriculum, in particular citing its social, personal and intrinsic worth. This endorsed the values expressed in the Cockcroft (1982) and Smith (2004) enquiries, and the National Curriculum (1999, 2007). They recognised that teaching should value each of the following learning outcomes though with a rather different emphasis to that currently seen in most classrooms:

- fluency in recalling facts and performing skills
- conceptual understanding and interpretations for representations
- strategies for investigation and problem solving
- awareness of the nature and values of the educational system
- appreciation of the power of mathematics in society.

Participants agreed that, in order to achieve these different learning outcomes, different classroom activities and teaching methods need to be deployed. These are described in the table, right.

[1] http://www.ncetm.org.uk

Learning outcomes sought	Types of learning activity implied
Fluency in recalling facts and performing skills	• Memorising names and notations • Practising algorithms and procedures for fluency and 'mastery'
Conceptual understanding and interpretations for representations	• Discriminating between examples and non-examples of concepts • Generating representations of concepts • Constructing networks of relationships between mathematical concepts • Interpreting and translating between representations of concepts
Strategies for investigation and problem solving	• Formulating situations and problems for investigation • Constructing, sharing, refining, and comparing strategies for exploration and solution • Monitoring one's own progress during problem solving and investigation • Interpreting, evaluating solutions and communicating results
Awareness of the nature and values of the educational system	• Recognising different purposes of learning mathematics • Developing appropriate strategies for learning/reviewing mathematics • Appreciating aspects of performance valued by the examination system
Appreciation of the power of mathematics in society	• Appreciating mathematics as human creativity (plus historical aspects) • Creating and critiquing 'mathematical models' of situations • Appreciating uses/abuses of mathematics in social contexts • Using mathematics to gain power over problems in one's own life

When participants were asked to compare their "vision for an ideal mathematics curriculum" with the values that are implied by the "curriculum that is currently implemented in most schools, colleges and other settings", a clear pattern emerged. Participants consistently reported that:

- too much time is spent developing "fluency in recalling facts and performing skills" to the detriment of other aspects
- much greater emphasis should be placed on the remaining four learning outcomes, with particular emphasis being placed on "conceptual understanding and interpretations for representations" and "strategies for investigation and problem solving".

There was remarkably little variation in these views either across phases of education or between the geographical areas where meetings were held.

2. *Principles for teaching*

Participants suggested amendments to the draft set of principles established at the initial conference and a final version is set out below. These principles have a sound basis in research.

Teaching is more effective when it:

- **Builds on the knowledge learners already have**
  Developing formative assessment techniques and adapting our teaching to accommodate individual learning needs.

- **Exposes and discusses common misconceptions and other surprising phenomena**
  Using learning activities that expose current thinking, create 'tensions' by confronting learners with inconsistencies and surprises, and allow opportunities for resolution through discussion.

- **Uses higher-order questions**
  Questioning is more effective in promoting mathematical thinking when it promotes explanation, application and synthesis rather than mere recall.

- **Makes appropriate use of whole-class interactive teaching, individual work and cooperative small group work**
  Collaborative group work is more effective after learners have been given an opportunity for individual reflection. Activities are more effective when they encourage critical, constructive discussion, rather than uncritical acceptance or argument for its own sake. Shared goals and group accountability are important.

- **Encourages reasoning rather than 'answer getting'**
  Often learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.

- **Uses rich, collaborative tasks**
  Tasks which promote mathematical learning are accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not' questions.

- **Creates connections between topics both within and beyond mathematics and with the real world**
  Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas.

- **Uses resources, including technology, in creative and appropriate ways**
  There are many rich resources that promote mathematical learning. For example, ICT offers new ways to engage with mathematics. At its best it is dynamic and visual: relationships become more tangible. ICT can provide feedback on actions and enhance interactivity and learner autonomy. Through its connectivity, ICT offers the means to access and share resources and – even more powerfully – the means by which learners can share their ideas within and across classrooms.

- **Confronts difficulties rather than seeks to avoid or pre-empt them**
  Effective teaching challenges learners and has high expectations of them. It does not seek to 'smooth the path' but creates realistic obstacles to be overcome. Confidence, persistence and learning are not attained solely through repeating successes, but by struggling with difficulties.

- **Develops mathematical language through communicative activities**
  Mathematics is a language that enables us to describe and model situations, think logically, frame and sustain arguments and communicate ideas with precision. Learners do not know mathematics until they can 'speak' it. Effective teaching therefore focuses on the communicative aspects of mathematics by developing oral and written mathematical language.

- **Recognises both what has been learned and also how it has been learned**
  What is to be learned cannot always be stated prior to the learning experience. After a learning event, however, it is important to reflect on the learning that has taken place, making this as explicit and memorable as possible. Effective teachers will also reflect on the ways in which learning has taken place, so that learners develop their own capacity to learn.

In Mathematics Matters, the NCETM invited participants to describe sample, illustrative lessons of 'best practice'. These may be found on the NCETM portal and serve to 'flesh out' these general principles.

3. Obstacles to progress

Participants identified many reasons for discrepancies between the current curriculum as implemented in schools and colleges and the curriculum they would wish to see. The most common reasons cited were:

- **Society's attitude towards mathematics**
  Society and its media still promote mathematics as a 'geeky' subject and mathematical incompetence remains socially acceptable.

- **Teachers' subject and pedagogical subject knowledge**
  Many teachers lack confidence in the subject and an awareness and understanding of appropriate approaches and resources (including ICT). They also lack time for continuing professional development.

- **A taught curriculum defined by assessment**
  Assessment should be defined by the curriculum aims, rather than the taught curriculum being defined by assessment. Teaching towards national tests and public examinations has led to an overemphasis on 'covering content' in a superficial manner, at the expense of developing deeper understanding and non-routine problem solving. This is, at least partly, due to the fragmented and artificial nature of many of these assessments.

- **The style and quality of textbooks and other resources**
  There is still an over-reliance on pre-packaged schemes in all phases. Many of these, particularly those targeting specific assessments, do not promote the types of learning educators value.

- **Initiatives that appear conflicting, disempowering and prescriptive**
  A simplistic interpretation of national strategies and the apparent inflexibility and mechanistic nature of inspection regimes (particularly internal ones), can lead to the production of externally acceptable forms of behaviour (e.g. 3-part lessons, learning objectives written on the board before the lesson) and inhibit principled, imaginative teaching. Teachers report that inconsistencies often appear between the practices sought by inspections internal to their organisations (often by non-specialists) and those that are recommended by outside agencies.

4. Suggested ways forward

Participants were also asked to suggest ways in which the above obstacles may be overcome. Recommendations centred around four themes, namely:

- **Improve the provision and quality of professional development opportunities**
  Participants proposed that continuing professional development should become an entitlement and expectation for all teachers. Effective, collaborative professional development should focus both on developing participants' subject and pedagogical knowledge, rather than on the mechanics of implementing the latest 'initiative'. Professional development should be characterised as reflective teachers researching their own practice and engaging with the research of others. Resources should be found to foster partnerships between institutions, particularly with those in HE.

- **Develop and share experience and resources for learning**
  Participants saw a need for the NCETM and others to begin to collate and exemplify the combined experience, wisdom and expertise of teachers of mathematics, so that these may be shared more widely. A range of resources were suggested, including: mathematical tasks and activities that encourage collaborative work and the development of mathematical processes; accounts and authentic videos of lessons that exhibit a wide range of effective practices.

- **Use professional standards to inform others about the teaching and learning of mathematics**
  Participants recommended that professional standards that describe and exemplify our values, principles and practices are developed and used to educate the perceptions of managers in schools and colleges so that more time may be made available for professional development and more informed performance assessment might take place. In addition, participants suggested that all those in mathematics education should continue to seek ways of promoting positive images of mathematics and mathematicians to society in general.

- **Influence the extent and quality of key stage assessments and public examinations**
  Participants generally felt that they wanted to see the elimination of, or at least a reduction in, the quantity of end of key-stage external assessment' and so create more time for teaching and learning, including formative assessment. They also felt that the content of these assessments should be revised to reflect the broader aims of mathematics teaching. Participants also felt that the 'high stakes' basis of league tables should be altered so that competition between schools is replaced by collaboration among teachers.

The NCETM will share all the underpinning evidence in the report with partners and stakeholders and will explore in greater depth the messages in the first three themes and use them to inform and influence its work.

Concluding remarks

Mathematics Matters is only a beginning. It is not suggested that the values and principles set out are in any way complete or exhaustive. Indeed, the NCETM intends that they will stimulate a wider, more informed debate. Further exemplification is essential if they are not to become just another collection of 'worthy words'.

The process has proved to be most stimulating and has offered a valuable professional development experience for many of those participating. In particular, the educators involved have reported how much they have valued the sharing of lesson accounts and reflecting on the values and principles these reveal. The National Centre therefore intends that this process will continue.

To date, the National Centre has collected 57 lesson accounts to illustrate the wide range of practices that illustrate the values and principles stated above. These accounts are too extensive to include in this report so they are available on the NCETM portal, categorised by:

- their underlying values
- the principles they illustrate
- the nature of the learner group.

The values and principles set out above are intended to encourage all those involved in mathematics education to be more explicit about their own priorities and goals and the types of task and classroom activity that they would wish to see. We strongly believe that it is possible for learners at every level to become excited about mathematics and about being mathematicians, particularly when they begin to experience the broader range of activities that are proposed.

RATIONALE

In 2007, the mathematics community celebrated two significant anniversaries.

It was 20 years since the publication of 'Better Mathematics' (Ahmed, 1987) and 25 years since the publication of 'Mathematics Counts' (Cockcroft, 1982). Each of these documents articulated views on what constitutes the effective learning of mathematics – informed by accumulated research findings and interpreted through the prevailing culture and values.

For example, in 'Better Mathematics' there is the memorable Statement 4:

> *Mathematics is effectively learned only by experimenting, questioning, reflecting, discovering, inventing and discussing. Thus, for children, mathematics should be a kind of learning which requires a minimum of factual knowledge and a great deal of experience in dealing with situations using particular kinds of thinking skills.*

And in 'Mathematics Counts' is found:

> *242 We wish now to discuss the implications of the previous sections for work in the classroom. We are aware that there are some teachers who would wish us to indicate a definitive style of teaching mathematics, but we do not believe that this is either desirable or possible. Approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities of both teachers and pupils. Because of the differences of personality and circumstance, methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with a different group of pupils. Nevertheless, we believe that there are certain elements which need to be present in successful mathematics teaching to pupils of all ages.*
>
> *243 Mathematics teaching at all levels should include opportunities for*
>
> - *exposition by the teacher;*
> - *discussion between teacher and pupils and between pupils themselves;*
> - *appropriate practical work;*
> - *consolidation and practice of fundamental skills and routines;*
> - *problem solving, including the application of mathematics to everyday situations;*
> - *investigational work.*
>
> *In setting out this list we are aware that we are not saying anything which has not already been said many times over many years. The list which we have given has appeared, by implication if not explicitly, in official reports, DES publications, HMI discussion papers and the journals and publications of the professional mathematics associations. Yet we are aware that although there are some classrooms in which the teaching includes, as a matter of course, all the elements we have listed, there are still many in which the mathematics teaching does not include even a majority of these elements.*

The NCETM framed the key question, "What constitutes the effective learning of mathematics?" on the understanding that the answers to this question are fundamental. Methods of teaching, the design of the curriculum, the use of assessment (both formative and summative), the organisation of learning groups, the selection and use of supporting resources and materials, the initial and continued professional development of teachers are all consequent on the answers to this question. It is appropriate that the question is asked by each generation and the answers refreshed in the light of experience and the latest research findings. Equally, the answers need to be interpreted through the prevailing culture and values.

Thus, the NCETM, with its clear brief to provide coordination and leadership on all aspects of professional development for teachers of mathematics, took responsibility for stimulating and undertaking a national debate in order to:

- establish an agreed set of valued learning outcomes and an agreed, and evidence-based, set of principles that underpins the teaching and learning of mathematics
- produce a collection of lesson accounts from a range of different settings in order to illustrate what the values and principles might look like in the classroom
- ascertain the relative emphasis put onto the different valued learning outcomes in an ideal case and to compare these with perceptions of relative emphasis in practice, and
- identify any barriers to the values and principles being translated into practice, and to propose strategies for overcoming them.

PROCESS

An initial conference, with 71 leading professionals in the field of mathematics education, was held in London in June 2007 to stimulate debate by:

- identifying, confirming and agreeing values and principles that underpin the effective teaching and learning of mathematics
- illustrating, through examples, how practice may reflect and interpret these core values and principles, and
- exploring the factors that inhibit or modify the implementation of these values and principles.

This was followed by a series of six one-day regional colloquia, held between September 07 and February 08 that were designed to:

- test levels of agreement with the values and principles articulated at the initial conference and to amend and refine them as appropriate, and to
- begin to build a collection of lesson accounts that illustrate what the values and principles may look like in practice.

These regional colloquia involved representation from early years and primary teachers, secondary teachers, teacher trainers and educators, adult education teachers (including prison), and the NCETM team and consultants. We recognise, of course, that this is not a representative sample, but rather an invited group of informed participants. In addition, we recognise the under-representation of early years and primary phase practitioners. The Centre has plans in place to address this matter.

The table below shows the location and timing of each colloquium.

Location	Date	Attendees
Leicester	28-09-07	13
Bath	18-10-07	11
Canterbury	02-11-07	14
Newcastle	11-12-07	11
Manchester	25-01-08	27
Peterborough	07-02-08	7

Each colloquium day started with an invitation to each participant to spend half an hour writing an account of a memorable, inspirational mathematics lesson, either taught or observed. This was followed by a discussion on the values and principles proposed at the initial conference, after which participants were invited to work together to respond in writing to these proposals, including their views on any differences between ideal and practised values. Next, participants returned to the lesson account, discussed it in depth with two other participants and then associated the account with the values and principles it best illustrated.

Excellent – wonderful to discuss teaching etc. for the day. A pleasure to listen to colleagues.

(P.Williams)

The afternoon started by identifying obstacles to teaching mathematics in accordance with the values and principles that had been identified. This was followed by group sessions that proposed strategies to overcome these obstacles.

Throughout the day, participants wrote their responses to all the activities on prepared templates (see Appendix 1), which facilitated subsequent collation and analyses. Participants had the option of having their lesson accounts ascribed or not.

Though the emphasis in each colloquium day was to gather information and views from participants, as opposed to provide information to them, their feedback overwhelmingly shows that participants found the day of personal, professional development value. They described time for reflection on their own practice and to discuss values and principles of mathematics education as among the most useful elements.

It was really good to have time to reflect and think about my own practice and the values and principles that underpin my own teaching.

(J.Knightbridge)

Values

The initial conference began by addressing the reasons why mathematics is considered to be important. Participants then tried to identify how these values reveal themselves in classroom practices. This, in turn, further led us all to reflect on how far current classroom practices are in accord with our values and possible reasons for any discrepancies.

In both the initial conference and subsequent colloquia, there was broad agreement on why mathematics is important. Our values echoed those found in, for example, the Cockcroft (1982) and Smith (2004) reports, the National Curriculum Programmes of Study (QCA, 2007). Three foci were identified:

Mathematics has value in enabling groups and societies to function effectively and creatively.

(R.Griffiths, P.Hough)

The social value of mathematics

Participants felt that mathematics continues to make a substantial contribution to the effective and creative functioning of our increasingly technological society, by equipping learners with the life skills they need at work and at home. Furthermore, mathematics can enable learners to develop the social skills they need for effective collaboration with others and to become aware of and appreciate the different perspectives they may have. Learners may also begin to develop an awareness of the historical-cultural-social origins of mathematics and its role in modern society.

> *Mathematics is of central importance to modern society. It provides the language and analytical tools underpinning much of our scientific and industrial research and development. Mathematical concepts, models and techniques are also key to many vital areas of the knowledge economy, including the finance and ICT industries. Mathematics is crucially important too, for the employment opportunities and achievements of individual citizens. (Smith, 2004 foreword, page v)*

> *Mathematical thinking is important for all members of a modern society as a habit of mind for its use in the workplace, business and finance; and for personal decision-making. Mathematics is fundamental to national prosperity in providing tools for understanding science, engineering, technology and economics. It is essential in public decision-making and for participation in the knowledge economy. (QCA, 2007)*

Mathematics enables learners to participate in life both at work and at home.

(J.Back)

The personal value of mathematics

Participants valued learners seeing themselves as mathematicians and appreciating the beauty of mathematics. They stressed the benefits of mental empowerment attained through cycles of abstraction and experience and the confidence gained when finding strategies to approach and solve problems. Mathematical learning was also felt to contribute to spiritual, moral and cultural development.

> *Mathematics equips pupils with uniquely powerful ways to describe, analyse and change the world. It can stimulate moments of pleasure and wonder for all pupils when they solve a problem for the first time, discover a more elegant solution, or notice hidden connections. Pupils who are functional in mathematics and financially capable are able to think independently in applied and abstract ways, and can reason, solve problems and assess risk. (QCA 2007)*

…learners seeing themselves as mathematicians.

(S.Feller)

The intrinsic value of mathematics

Participants valued mathematics as a creative discipline in its own right. They noted that mathematics offers learners a uniquely powerful set of tools with which they can interpret, organise and sort information, analyse and solve problems and communicate with others. Mathematics was also considered to be the 'mother of all languages', a grammatology of thinking and a way of making meanings, viewing and describing the world.

> *Mathematics is a creative discipline. The language of mathematics is international. The subject transcends cultural boundaries and its importance is universally recognised. Mathematics has developed over time as a means of solving problems and also for its own sake. (QCA 2007)*

…confidence in strategies to approach and solve problems.

(J.Golding, B.Murphy)

Types of learning outcome and classroom activities implied by these values

At the initial conference there was also considerable agreement as to the type of learning outcomes valued and the different types of classroom activity that these outcomes might imply. The following non-hierarchical list was found helpful:

> ***…to organise and sort information, to solve problems and explore*** **the world.**
>
> **(R.Hyde)**

Learning outcomes sought	Types of learning activity implied
Fluency in recalling facts and performing skills	• Memorising names and notations • Practising algorithms and procedures for fluency and 'mastery'
Conceptual understanding and interpretations for representations	• Discriminating between examples and non-examples of concepts • Generating representations of concepts • Constructing networks of relationships between mathematical concepts • Interpreting and translating between representations of concepts
Strategies for investigation and problem solving	• Formulating situations and problems for investigation • Constructing, sharing, refining, and comparing strategies for exploration and solution • Monitoring one's own progress during problem solving and investigation • Interpreting, evaluating solutions and communicating results
Awareness of the nature and values of the educational system	• Recognising different purposes of learning mathematics • Developing appropriate strategies for learning/ reviewing mathematics • Appreciating aspects of performance valued by the examination system
Appreciation of the power of mathematics in society	• Appreciating mathematics as human creativity (plus historical aspects) • Creating and critiquing 'mathematical models' of situations • Appreciating uses/abuses of mathematics in social contexts • Using mathematics to gain power over problems in one's own life

Source: (Swan, 2006 p.42)

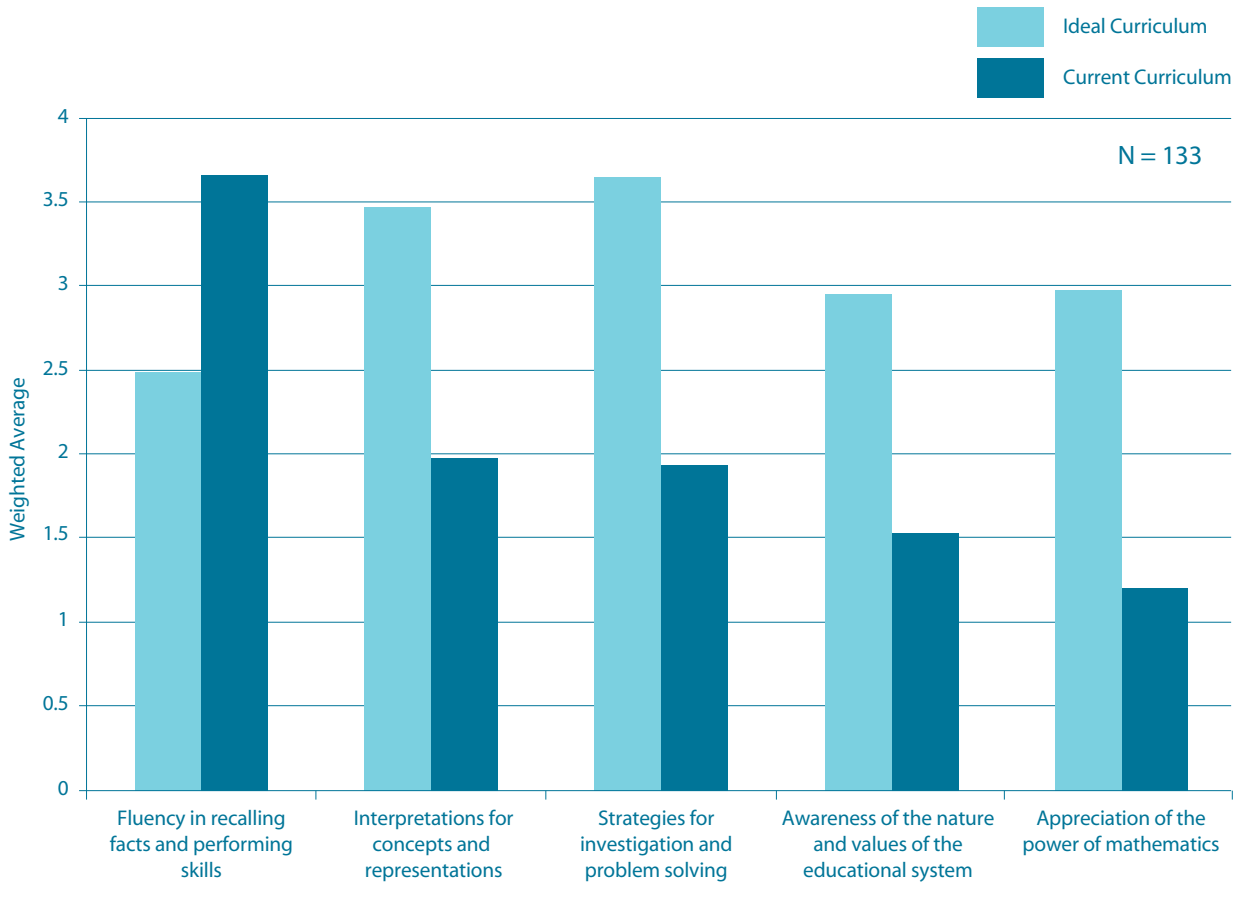
> ***…a way of making meanings, of viewing and describing the world.***
>
> **(P.Lacey, P.Drake)**

Comparing ideal values with values in practice

When participants were asked to compare their "vision for an ideal mathematics curriculum" with the values that are implied by the "curriculum that is currently implemented in most schools and other settings", a consistent pattern emerged. Participants recorded on a four-point scale how frequently mathematics lessons should include each learning outcome and also how frequently mathematics lessons, in their view, actually do reflect each learning outcome (1 = hardly ever, 4 = almost every lesson). The aggregated responses of 133 initial conference and colloquia participants are shown in the graph below.

The results show that participants consider that fluency is overvalued in the current curriculum, while the remaining values are undervalued. Perhaps surprisingly, this pattern of response was repeated consistently across responses from each phase of education (primary, secondary and tertiary) and across all regions. We note again, however, that due to the small and self-selecting nature of the sample involved (particularly with regard to primary teachers) these results may not reflect those of the profession as a whole.

All respondents: Ideal and Current Outcomes – Weighted Average where high = most important



Illustrating different types of learning outcome

Worthwhile learning tasks are likely to reflect more than one of the purposes set out above, but there may be a tension if teaching attempts to serve incompatible purposes at the same time. For example, if learners are asked to 'investigate' an open situation, while the teacher wants them to 'discover' a culturally valued result (such as Pythagoras' theorem), an incompatibility arises between the divergence of the learners' explorations and the convergent purposes of the teacher.

Bearing this in mind, participants at the initial conference were invited to describe inspiring examples of lessons they had seen and identify the purposes being served. Below, we offer five of their contributions, and identify each one with a different purpose. Some of the contributions relate to specific phases, though some of the ideas may be adapted for use in other phases.

Example 1.
Fluency in recalling facts and performing skills (A. Watson)

Learners constructing examples introduces a sense of the structure, characteristics and properties of new objects. This example, from a secondary classroom, also includes practice and fluency. The 'big' idea of conjugation is being introduced building on successful past tool use of the grid. There is a sense of ownership, mystery, intrigue. Also it feels do-able, but the difficulty adds to the motivation. Also, you don't have to tell learners everything – or show them.

What was the mathematical task?
Having been just introduced to the idea that there are numbers of the form $a + \sqrt{b}$, and reminded of grid multiplication, learners were asked to find two such numbers which, when multiplied, you 'lose' the root part.

x	a	$\sqrt{b}$
c		
$\sqrt{d}$		

Learners worked on their own or in pairs. All rough working was kept and it was clear that most learners shifted from some sort of 'testing' various integers' approach to a 'structural' approach, e.g.: trying $(2+\sqrt{3})(3+\sqrt{2})$ or $(2+\sqrt{2})(2+\sqrt{2})$ etc. Many started by using calculator, some abandoned this as they focused on structure.

What learning culture was created? How was this achieved?
Learners trusted the teacher because of having done similar tasks in the past. They were allowed to choose starting numbers and methods of working, except for grid multiplication. There was lots of talk. Some 'gossiped' the method (distributed knowledge!). Teacher asked 'why choose this, why choose that?' and reminded them of purpose. Some of the learners carried on after class together or separately and the teacher was interested in their work!

How could you tell that the task achieved the intended purposes?
I saw the written work. Some aspects of purpose achieved by all: $(a+\sqrt{b})(a-\sqrt{b})$ = integer found by some.

Is this example available to see/read about?
It will be (special issue of Educational Studies in Mathematics 2008)

Can you say why you chose this example? What criteria were in your mind?
To illustrate use of learner-generated examples and to promote shifts between empirical and structural reasoning.

Example 2.
Interpretations for concepts and representations (P. Lacey)

Extrapolating new knowledge from old is an engine for exploring and charting the territory of mathematics. In this exercise learners are expected to apply their understanding of fundamental concepts and principles (inverse, equivalence etc) in order to 'map' and extend their knowledge.

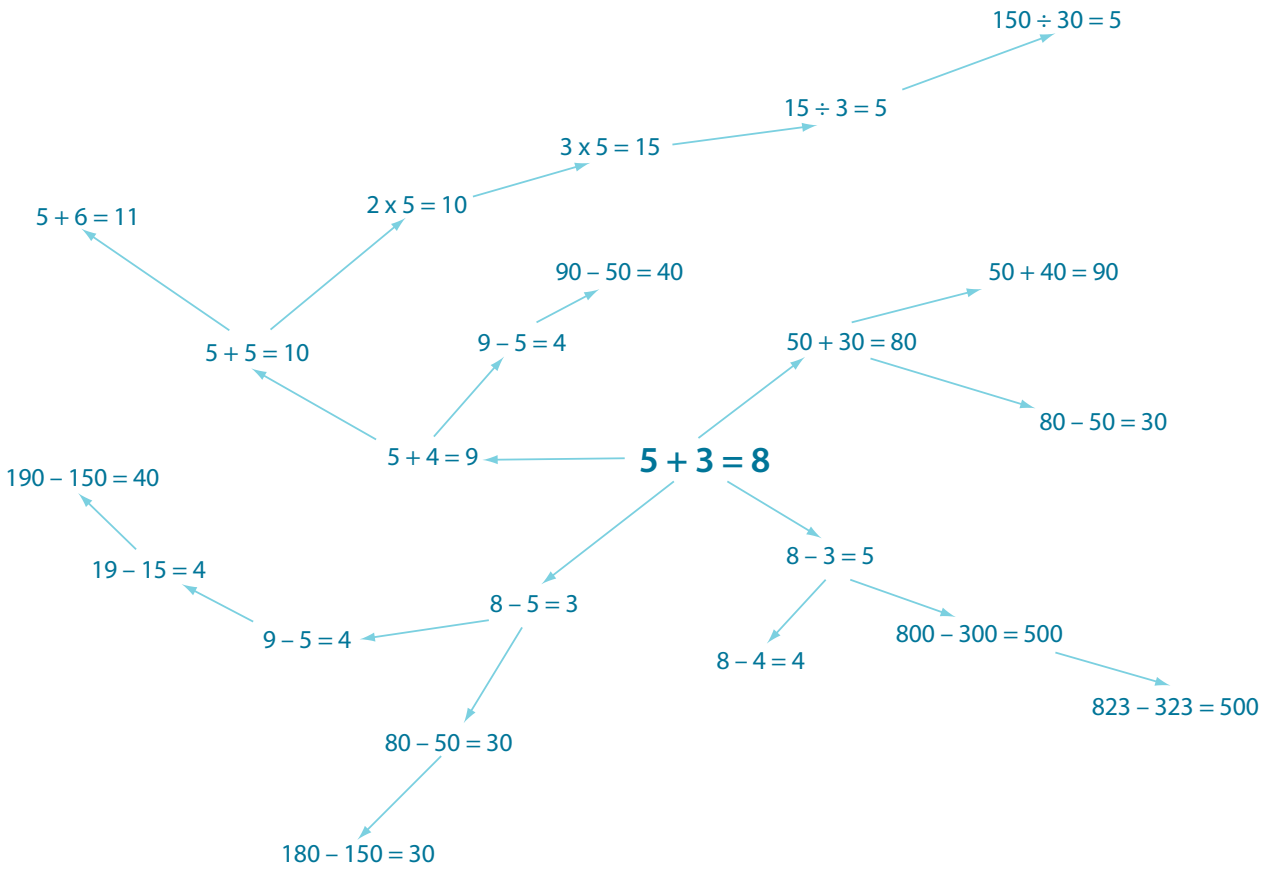
What was the mathematical task?
What else do you know (and why) if you know that 5 + 3 = 8? The statement was written in the middle of a board visible to a whole class. Initially whole class responses. For example: 500 + 300 = 800. This was written on the board with an arrow to it from the original statement. A 'mind-map' was generated with answers to the 'why' bit of the question determining the connections. This lead on to learners working in groups/pairs to create a 'composite' mind map. Discussing the 'why' proved productive.

What learning culture was created? How was this achieved?
Learners were in control. There was no perceived limit. There was genuine sharing of personal understandings.

How could you tell that the task achieved the intended purposes?
I think the activity actually altered the views of some of the learners on what mathematics actually is. Certainly challenged the 'quantum' view of mathematics as isolated facts.

Is this example available to see/read about?
Reported in 'Mathematics Teaching' 187 June 2004 as a special conference insert, after being included in an ATM annual conference presentation.

Can you say why you chose this example? What criteria were in your mind?
Simple and accessible start – almost trivial; but deep in its engagement. Explicit and shared discussions on personal maps of understanding have a sense of deep learning.

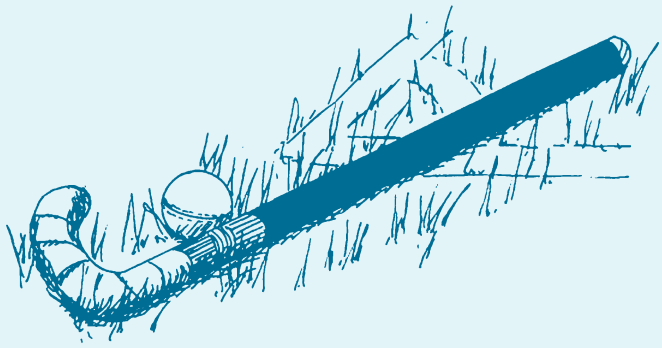


Example 3.
Strategies for investigation and problem solving
(J. Edwards)

What was the mathematical task?

FINAL SCORE

The final score in a hockey game was 2 – 1.
List the possible half-time scores.
How many possible scores are there?
Try this for other scores.



From: Graded Assessment in Mathematics (GAIM) Macmillan Education 1988

The GCSE class was set 10 of 10, a group of 3 girls. In the previous lesson, learners had generated enough data to generalise and after considerable debate, they arrived at the number of possible half-time scores being (x+1)(y+1) where x and y are the final scores. When challenged to explain why this expression 'worked', they argued, challenged, justified amongst themselves for half an hour. Eventually, they were able to describe why they needed to add one to each score. The value of this is was in: (a) being able to comfortably challenge each other; (b) achieving 'hard' maths (for them); (c) being aware that mathematics can describe situations d) being able to interpret a situation mathematically.

What learning culture was created? How was this achieved?
A background of feminist epistemology.
The 10 school 'classroom rules' were abandoned in favour of 3 mathematics classroom rules:

- *everyone does mathematics*
- *everyone does mathematics in a way that enables others to do mathematics*
- *everyone shares their mathematics (collaborates).*

Small groups were based on friendship, there was an ethos of collaboration, feminist principles of connectedness.
A shared understanding that a mathematics classroom is where thinking takes place.

How could you tell that the task achieved the intended purposes?
Being able to communicate the outcomes and process both verbally and in writing. Pleasure expressed by the pupils about their achievement. (Despite being in set 10 of 10, this 'diet' of mathematics learning resulted in GCSE grades D,E,E for this group of girls.)

Is this example available to see/read about?
Graded Assessment in Mathematics (GAIM) Macmillan Education 1988 p68.

Can you say why you chose this example? What criteria were in your mind?
Level of enjoyment for pupils;
Level of challenge;
Potential for awareness that mathematics can describe 'real' situations;
Potential to develop confidence – 'doing quadratics' is for 'good' people.

Example 4.
Awareness of the nature and values of the educational system
(M. Swan)

This task was designed to help learners become aware of what it feels like to teach something, and in so doing to review and reorganise their own understanding of a topic.

What was the mathematical task?
Two parallel year 7 classes were taught different topics by different teachers. One class was taught "Area" and the other was taught "Bearings". At the end of this period, the teachers told their learners that they would now be given two lessons to prepare to teach the topic they had just learned to members of the other class.

Learners were given some advice on how to go about this process. They were asked to decide exactly what the big ideas in the topic were, the order these should be presented in, to identify difficult ideas and prepare resources help to explain them (e.g. a worksheet). They were also asked to think how they would find out if their teaching had been successful. When both classes were ready, half of the pupils moved to the other classroom and each one sat alongside a pupil from the other group and taught them the topic. In a second lesson, roles were reversed.

What learning culture was created? How was this achieved?
Learners appeared nervous but very motivated by this challenge. They took a great deal of trouble in creating attractive and informative worksheets and some even had them typed out and checked beforehand. During the teaching phase, learners adopted teacher roles with some enthusiasm. One girl (teaching two boys due to uneven class sizes), adopted a very confident style, making statements like:

"How do you find the area from these two numbers? You can draw centimetre squares if you want."
" Talk then or you won't learn anything!"
"Pretend you are the teacher, how would you explain it to me?"
"Put your hands up if you don't know."

The boys, it must be said, couldn't wait for their turn to teach! This positive culture was achieved because these teachers had high expectations for their classes and wanted their learners to take active roles in lessons.

How could you tell that the task achieved the intended purposes? Do you have any evidence?
We didn't collect evidence on the mathematics learned, but the reflective thought that went into the review and design of the teaching sessions was considerable. Pupils began to show evidence of appreciating that teaching is not just about 'telling'. When, for example, the girl above was asked about the experience, she responded as follows: (Int = interviewer):

Int: What was hard about being a teacher?
Girl: After teaching for some time, I find they don't understand.
Int: How did the experience help you?
Girl: It gave me more confidence.
Int: Would you like to try teaching again?
Girl: Yes, so what I've learned now I must try on other pupils so I get better at teaching. I can't understand how to teach well.

Is this example available to see/read about?
This is referred to in (Bell, A; Crust, R; Shannon, A; Swan, M; "Awareness of Learning, Reflection and Transfer in School Mathematics", ESRC Project report: R000-23-2329, Shell Centre for Mathematical Education, University of Nottingham)

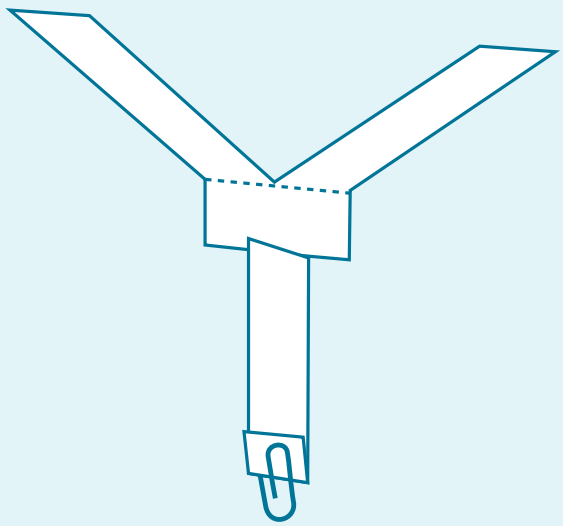
Can you say why you chose this example? What criteria were in your mind?
To illustrate how changing classroom roles can begin to develop pupils' awareness of the nature of teaching and learning itself. This can be done alongside the development of mathematical skills and concepts.

Example 5.
Appreciation of the power of mathematics in society (D. Pratt)

What was the mathematical task?

- *What makes the best design for a paper 'helicopter'?*
- *Make a spinner and try it out.*
- *Compare 2 spinners.*
- *Why did you like that one?*
- *What variables affect the flight (usually time of flight)*
- *In groups, explore the variable (e.g. length of wings) comparing it to the dependent variable (time).*
- *Perhaps use Active Graphing:*

(a) Enter a few ordinate points into a spreadsheet
(b) Generate a scatter-graph
(c) Decide what to do next in the experiment
(d) Go to (a).



What learning culture was created? How was this achieved?

No one task generates a culture. Indeed, a task can create discipline problems if the expectations of behaviour in the task clash with the prevailing culture. In this task, we were aiming for a culture in which mathematics is used to explore an interesting problem, resulting in the learning of new mathematics. Learners needed to work in small groups and work together, sharing roles. The mathematics was seen as powerful.

How could you tell that the task achieved the intended purposes?

Learners:

- *used graphs to make decision about where to take the experiment*
- *used data analysis methods to increase accuracy of experiment*
- *worked productively together*
- *could describe relationships between the independent and dependent variables.*

Is this example available to see/read about?

Ainley, J., Pratt, D. and Hansen, A. Connecting engagement and focus in pedagogic task design; British Educational Research Journal. 32.1, 23-38. 2006.

Ainley, J., Pratt, D., and Nardi, E; Normalising: children's activity to construct meanings for trend; Educational Studies in Mathematics, 45, 131-146. 2001.

Ainley, J. Super 'copters Micromath Autumn 1991.

Can you say why you chose this example? What criteria were in your mind?

Engaging learners (purpose); Engagement leading to focus (utility); Problem solving; Awareness of the power of mathematics (graphing as an analytical tool; average for smoothing errors). I have used the example many times and at many levels. It seems to provide an opportunity for thinking of graphs as analytical tools, rather than as presentational tools. Chosen out of frustration at narrow use of graphing in schools.

At the beginning of each colloquium, participants were asked to describe a lesson that they had found memorable and inspiring. In total, we received 57 lesson accounts, which have been checked and validated and are now available on the NCETM portal **www.ncetm.org.uk** Each is categorised in the following ways:

1. By the values that underpin the purpose of it.

2. By the principles being demonstrated.

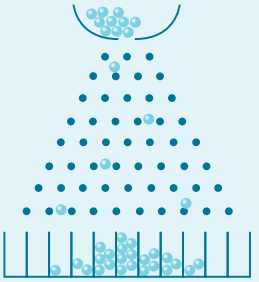
3. By the nature of the learner group.

A sample lesson account is given below. The teacher has classified this as developing "strategies for investigation and problem solving"; and as illustrating "encourages reasoning rather than 'answer getting'" and "creates connections between topics" for 11–16.

Name: Dan Curran	**Organisation:** Norton College

Age/ability range
Year 7 class mixed boys and girls.
The students were studying probability concepts both theoretically and experimentally.

How was the session/task introduced?
I had a slide of the old-fashioned 'pinball' type toy where a ball can be dropped in from the top and the ball makes its way to bins at the bottom. Several students recognised this type of equipment and we discussed what might happen to balls as they were dropped into the pinball machine. The discussion was good with various ideas about how the bins might fill up; an equal distribution of balls in the slots was a common thought.



How was the session/task sustained?
After generating interest in the outcomes, I showed them a virtual pinball simulator available as a Java type animation. (Web address of applet: **http://www.jcu.edu/math/isep/Quincunx/Quincunx.html**)

We experimented with the virtual pinball as a whole class activity, dropping balls of various sample sizes and with different numbers of bins at the bottom of the machine. A helpful visual feature of the applet was the generation of a frequency diagram revealing the way in which the balls fell.

The distribution of balls wasn't really what the class had generally expected. Claims of bad luck and fluke results were a general explanation for the distributions generated, so we re-ran the pinball simulator several times.

Amongst the class there was much debate about why the results were not as expected. Some students began to venture explanations for the distribution and started to talk about the 'routes' that led to the bins at the bottom of the pinball machine.

How was the session/task concluded?
Students suggested we reduce the simulator to a small number of bins and we then attempted to analyse the ways in which the balls could reach the bins. Within a short space of time, explanations for the results yielded by the simulator were produced. We verified our thoughts with some attempts to predict the number of balls our theoretical models were predicting.

What were the critical moments?
When the students started to "list" the possible routes to the slots, a connection to the structure of the pinball machine became apparent. They generated the routes themselves – albeit this was often attempted in a haphazard and unstructured way.

A second feature was the notion of experimental probability compared with the theoretical model. Lots of good discussion about ideas of luck and chance influencing deviations from our expectations. The ability to verify or discount ideas quickly made the debate manageable.

What mathematics was learnt?
Apart from the theoretical and experimental probability ideas, there were opportunities for discussion of how the results were being represented graphically. An unexpected bonus was that the class was able to consider very large samples.

How was that mathematics learnt?
Mainly discussion and recording the 'routes' through the pinball machine.

Resources
The lesson was displayed through the data projector and interactive whiteboard.

Principles that guide teaching

In articulating their values, participants began to clarify the learning goals and classroom activities they seek to achieve. They now attempted to identify research-based principles that could be used to inform the way they teach towards these goals.

Different goals require different types of learning activity and an appropriate choice of teaching strategy. For example, in order to achieve fluency in the recall of facts or the performance of skills, it is appropriate to emphasise personal practice. In order to develop conceptual understanding however, the teacher must encourage learners to notice 'sameness' and 'difference', examine alternative representations and share and discuss interpretations. In order to develop problem solving strategies, teachers need to allow learners to compare the outcomes of applying different strategies to an unstructured problem and examine the advantages and disadvantages of each. Different goals clearly make different pedagogical demands on a teacher.

Principles for effective teaching therefore depend on the purposes they seek to address. The recent "Improving Learning in Mathematics" materials, focused on the conceptual development and sought to embody the following list of research-based principles (DfES, 2005).

Teaching is more effective when it…	
1. Builds on the knowledge learners already have	This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs.
2. Exposes and discusses common misconceptions	Learning activities should exposing current thinking, create 'tensions' by confronting learners with inconsistencies, and allow opportunities for resolution through discussion.
3. Uses higher-order questions	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall.
4. Uses cooperative small group work	Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important.
5. Encourages reasoning rather than 'answer getting'	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.
6. Uses rich, collaborative tasks	The tasks used should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not' questions.
7. Creates connections between topics	Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas.
8. Uses technology in appropriate ways	Computers, interactive whiteboards and graphics calculators offer learners opportunities to explore rich 'microworlds', and to represent, analyse and communicate mathematics with interactive, dynamic tools.

Teaching is more effective when it develops awareness of the 'bigger picture'.

(M.Horner, R.Hyde, J.Garvey, Jones, P.Lacey, Taylor, G.Wake)

At the colloquia, 64% (46/72) of participants totally agreed with the principles presented, 32% mainly agreed, expressing reservations, while 4% expressed particular concerns. No participants stated that they disagreed with the principles.

Of those who expressed some reservation or concern, 35% (9/26) related to the use of technology – it should be used as a tool and is not always necessary. Colleagues working outside traditional educational settings pointed out that technology is not always available. 23% cited concerns about the confusion that may be caused when exposing and discussing common misconceptions (R.Archer, J.Bean, M.Dovey).

Participants proposed a number of amendments to the principles, around using whole class interactive teaching; creating connections within mathematics and in the real world; and using all resources creatively .

Participants proposed some additions to the principles around confronting difficulties; developing mathematical language; and recognising learning.

Thus, we have retained all the original principles, with slight rewording to 2, 4, 7, 8:

Teaching is more effective when it…	
1. Builds on the knowledge learners already have	This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs.
2. Exposes and discusses common misconceptions and other surprising phenomena	Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies and surprises, and allow opportunities for resolution through discussion.
3. Uses higher-order questions	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall.
4. Makes appropriate use of whole class interactive teaching, individual work and cooperative small group work	Collaborative group work is more effective after learners have been given an opportunity for individual reflection. Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important.
5. Encourages reasoning rather than 'answer getting'	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.
6. Uses rich, collaborative tasks	The tasks used should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and what if not' questions.
7. Creates connections between topics both within and beyond mathematics and with the real world	Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas.
8. Uses resources, including technology, in creative and appropriate ways	ICT offers new ways to engage with mathematics. At its best it is dynamic and visual: relationships become more tangible. ICT can provide feedback on actions and enhance interactivity and learner autonomy. Through its connectivity, ICT offers the means to access and share resources and – even more powerfully – the means by which learners can share their ideas within and across classrooms.

We have also added the following principles, which we believe can also be substantiated from research:

Teaching is more effective when it…	
9. Confronts difficulties rather than seeks to avoid or pre-empt them	Effective teaching challenges learners and has high expectations of them. It does not seek to 'smooth the path' but creates realistic obstacles to be overcome. Confidence, persistence and learning are not attained through repeating successes, but by struggling with difficulties.
10. Develops mathematical language through communicative activities	Mathematics is a language that enables us to describe and model situations, think logically, frame and sustain arguments and communicate ideas with precision. Learners do not know mathematics until they can 'speak' it. Effective teaching therefore focuses on the communicative aspects of mathematics by developing oral and written mathematical language.
11. Recognises both what has been learned and also how it has been learned	What is to be learned cannot always be stated prior to the learning experience. After a learning event, however, it is important to reflect on the learning that has taken place, making this as explicit and memorable as possible. Effective teachers will also reflect on the ways in which learning has taken place, so that learners develop their own capacity to learn.

Teaching is more effective when it develops mathematical language through communicative activities.

(B.Vertes)

Unhelpful principles

We invited participants to describe any principles that they believe are commonly held, but which are positively unhelpful to teachers.

Participants generated the following list of what they considered to be unhelpful advice:

- *Learn how to do it first – understanding will come later.*
- *Repetition will improve understanding.*
- *There is a 'best way' to teach, an 'optimal sequence' for learning, a 'right way' to solve each problem.*
- *Explain clearly how to do the problem before you give it to your class. Learning must be preceded by instruction.*
- *Tell the class your lesson objectives at the beginning of each lesson.*
- *Make sure your lesson has three parts.*
- *Plan the plenary discussion very carefully and stick to your plan.*
- *Cover the syllabus, even if this means hurrying through some parts.*
- *Keep emphasising presentation and neatness.*
- *Knowing the answer is important.*
- *Keep learners busy. Learners go off-task if they talk.*
- *Don't confuse learners by showing them incorrect methods.*
- *Use technology whenever you can.*

Most of these 'unhelpful principles' are well-intentioned but they become unhelpful when their continued emphasis encourages superficial activity rather than deeper learning.

Mathematics teachers have to overcome society's attitudes:

- ***that mathematics is "geeky" rather than "cool"***
- ***that it is acceptable not to be good at mathematics.***

(P.Williams, H.Pomroy, E.Lawrenson, L.Bell, M.Thomas, M.Elliot)

Obstacles to progress

At the initial conference and the subsequent colloquia, participants were invited to respond to the two open questions:

What are the major obstacles to progress?
How do these obstacles function?

These questions unlocked some strong feelings among participants. The responses are categorised into five overlapping themes, below. It must be emphasised that these themes were not put to participants, they emerged from the responses.

1. Society's attitude towards mathematics

Participants felt that society and its media have a limited, narrow view of mathematics and still promote it as a subject for 'geeks'. Mathematical incompetence continues to remain socially acceptable. Some also commented on the climate of distrust of teachers and the excessively detailed accountability that permeates and distorts the teaching of mathematics.

2. Teachers' subject and pedagogical subject knowledge

Participants felt that many teachers lack confidence in the subject and an awareness and understanding of appropriate resources (including ICT). They also lack time for continuing professional development. Participants commented that many teachers 'lack confidence' in mathematics, 'can't see the bigger, interconnected picture' and 'can only see one learning pathway through the curriculum'. When unexpected insights, answers or misconceptions arise, for example, many teachers don't have sufficient subject knowledge to depart from their predetermined plan. This is also evidenced by the over-dependence on pre-packaged schemes and textbooks.

One participant asserted that a common view of mathematics shared by teachers and pupils is that it is a 'dead weight of predetermined knowledge to be absorbed', rather than an organic system to explore collaboratively. In attempting to deal with learner disaffection, participants felt that many reward systems encourage extrinsic motivation, rather than an intrinsic satisfaction in the subject itself. They also felt that there is a developing practice of over-praising or praising too soon and teachers being trained to control, not empower.

Compounding this, it was felt that there is a lack of subject-specific CPD within schools and colleges, with opportunities for research and reflection.

Teachers' lack of confidence in mathematics can hinder 'letting go' of the content.
(P.Drake)

This section is concluded with one quote from the adult education sector that illustrates how the above factors can act together to impede learning:

> *In adult education settings, the wide range of learner interests and abilities, the likelihood of learners aiming for different qualifications, the sometimes variable attendance patterns, the typical non-resource rich learning environments, all set in the context of funding by results, present challenges to teachers that detract from the core business of igniting learner curiosity, building learner confidence and assuring learner achievement in mathematics.*
>
> *(S.Bertenshaw, I.Brazewell, C.Cairns, N.Culverwell*

There is a lack of subject specific training in schools.
(J.Jones)

There is a lack of high-quality, extended CPD opportunities with time for research and learning from reflection.
(V.Brown, W.Wilson)

3. A taught curriculum defined by assessment
Participants felt that assessment should be defined by the curriculum aims, rather than the taught curriculum being defined by assessment. Teaching towards national tests and public examinations has led to an overemphasis on 'covering content' in a procedural manner, almost to the exclusion of developing deeper relational understandings and problem-solving strategies. This is at least partly due to the fragmented and artificial nature of many of these assessments. Current formative and summative assessment strategies may thus be said to obstruct effective pedagogies. Participants asserted that it will be impossible to make meaningful improvements to teaching mathematics if assessment continues to be uninspiring and defined by what is measurable rather than what is valued.

High stakes external testing leads teachers to aim at short-term instrumental understanding.
(H.Tanner)

The aim is seen as passing tests rather than learning mathematics.
(P.Griffin)

The publication of examination figures, it was asserted, has lead to short-term initiatives being imposed on teachers, and a reluctance to take risks and innovate. The accompanying excessive paperwork and administration has further detracted from the job of teaching.

It should be noted that participants broadly welcomed the new Programmes of Study, with their renewed emphasis on the 'Key Concepts' and 'Key Processes'. These, it was hoped, would have a significant positive influence on the nature of national testing.

Assessment strategies do not reflect effective pedagogy.
(K.Jones)

It will be impossible to make meaningful improvements to teaching mathematics if assessment continues to be uninspiring – demanding the regurgitation of facts and 'teaching to the test'.
(H.Isherwood)

4. The style and quality of textbooks and other resources

Participants felt that there is still an over-reliance on pre-packaged schemes. Most do not promote the types of learning they value. Participants commented that textbooks, usually associated with awarding bodies, are major obstacles to effective practices, because they atomise the curriculum and promote teaching to the tests. They also commented on teachers' continuing lack of confidence in using ICT.

Some participants expressed a wish to see the access to resources improved, perhaps by the development of a bank of rich tasks. Others however, again commented that teachers have little time to explore new resources with colleagues.

Textbooks discourage connection making.

(R.Deakin, S.Wall, M.Colley)

Many teachers lack confidence in using ICT for teaching and learning.

(R.Deakin, P.Williams, M.Colley)

Revision books condense content that learners have never had chance to explore before condensing.

(P.Lacey).

There is a culture of unthinking teacher conformity rather than thoughtful experimentation.

(P.Griffin)

Fear of testing leads teachers to teach to the test; fear of authority keeps lessons safe and dull.

(J.Rickwood)

Teachers have misconceptions about what they are 'allowed' to do.

(K.Cross)

5. Initiatives that appear conflicting, disempowering and prescriptive

Participants felt that a simplistic interpretation of the national strategies and the apparent inflexibility and mechanistic nature of inspection regimes (particularly internal ones), can lead to the production of externally acceptable forms of behaviour (e.g. 3-part lessons, learning objectives written on the board before the lesson) and inhibit principled, imaginative teaching.

Participants commented that a major obstacle has been that teachers adopt local interpretations about what they are 'allowed' to do and 'fear stepping out of line'. This tendency is reinforced by senior managers who lack awareness of effective teaching practices in mathematics. This can result in a culture of unthinking teacher conformity rather than one of thoughtful, creative experimentation.

It was felt that, whatever the intentions of inspection and/or changes to the strategies, many teachers still believe that they have to teach in a prescribed way and this has led to a feeling of disempowerment which is compounded by frequent changes to national tests and public examinations.

Strategies for overcoming obstacles

At the initial conference, in answer to the question, "What practical steps can we take to help ourselves and others to overcome these obstacles?" there were high levels of agreement. Recommendations centred around four broad themes, namely:

1. Improve the provision and quality of professional development opportunities

Participants proposed that continuing professional development should become a career-long entitlement and expectation for all teachers. They thought that effective, collaborative professional development should focus on developing participants' subject and pedagogical knowledge, rather than on the mechanics of implementing the latest 'initiative'. Professional development should become characterised by reflective teachers researching their own practice and in engaging with the research of others.

Resources should be found to foster partnerships between institutions, particularly with those in HE. Teachers should be encouraged to develop through extended Masters level courses.

Participants said that they needed a wide range of settings in which they can question, enquire, critique their own practices and those of others. Simple practical possibilities include timetabling an hour a week to stop and read a professional journal or explore a website; cultivating a critical friend to develop materials and ideas with; and taking every opportunity to see the practice of others. More ambitious suggestions included the formation of lesson study groups; and groups which meet to develop, trial and report back on new tasks for the classroom.

Ensuring that continuous teacher learning is a professional requirement and entitlement for teachers of mathematics in all settings.

Creating opportunities for professional development within and across learning settings.

(A.Brunt, L.McClure, P.Wylie, N.Honey, P.Hough)

Using coaching to encourage risk taking and innovation by teachers.

(J.Bean, A.Brunt, J.Pattison, R.Sutton)

2. Develop and share experience and resources for learning

Participants saw a need for the NCETM and others to collate and exemplify the combined experience, wisdom and expertise of teachers of mathematics, so that these may be shared more widely. A range of resources were suggested, including:

- mathematical tasks and activities that encourage collaborative work and the development of mathematical processes
- accounts and authentic videos of lessons that exhibit a wide range of effective practices
- empirical evidence and case studies that describe effective approaches and illustrate their effects on examination results
- a resource bank of real-life applications of mathematics
- a glossary of mathematical terms
- a compendium of proven and successful lessons/tasks
- teacher discussion areas.

It was recognised that some of these aspects are already being developed on the NCETM portal (e.g. through the discussion boards and the Mathemapedia) and through the Bowland Maths initiative website, but it was also noted that teachers need additional free time to absorb and work with such resources.

Give teachers time to develop ideas of ways of teaching and the resources to help them.

(H.Isherwood)

Provide convincing stories to demonstrate that learners can both learn more effectively through active engagement and pass exams.

(D.Wright)

As an inclusive community, agree what we want mathematics teaching and learning in classrooms to look like.

(G.Wake)

3. Use professional standards to inform others about the teaching and learning of mathematics

Conference participants recommended that professional standards that describe and exemplify our values, principles and practices are developed and used to educate the perceptions of managers in schools and colleges so that more time may be made available for PD and more informed performance assessment might take place. It should be noted that during the time of the Mathematics Matters project, the NCETM provided some mathematical exemplification of the professional standards for teaching, as well as guidance for managers in using these in performance reviews.

In addition, participants in the project suggested that ways of promoting positive images of mathematics and mathematicians to society in general should continue to be sought – perhaps through sport, media, music, popular role models, family mathematics days, family homeworks, roadshows, etc.

Participants were clear that we must work to remove common misinterpretations of national guidance, particularly those that emphasise form over substance, such as the "expectation of a 3-part lesson". Some participants suggested that it would be helpful to provide an authoritative document that would enable teachers to explicitly address these misinterpretations with senior managers. Others suggested using the NCETM exemplification of the professional standards to inform performance management, including professional and career development. It was also recognised that heads of department need support as they fight for the time and resources they need in order to conduct effective planning and CPD.

> ***Teachers need to be given permission to do what they know leads to good learning – even if it doesn't always fit with perceived expectations.***
>
> **(R.Hyde)**

4. Influence the extent and quality of key stage assessments and public examinations

Participants expressed many concerns about testing and generally felt the elimination of or at least reduction in the influence of end of key-stage external assessment was desirable, so creating more time for teaching and learning. A number of suggestions were made which participants felt would change the focus away from tests and onto teaching and learning. In particular, they proposed a greater focus on formative assessment by teachers and that the content of summative assessments should be revised to reflect the broader aims of mathematics teaching.

> ***Use the NCETM exemplification of the professional standards to help teachers and managers understand what is expected of mathematics teachers.***
>
> **(R.Tanner)**

Participants also wanted to see teachers supported to develop professional confidence that good teaching practices can be employed without a detrimental effect on the performance in high status assessment; there exists clear and accessible evidence of good test and examination results as a consequence of not "teaching to the test".

> ***Somehow downgrade the importance of the exams or at least have broader measures of assessment.***
>
> **(M.Homer)**

> ***Define measures of progress that relate to depth rather than only coverage.***
>
> **(M.Dovey**

> ***Support/reinstate the culture of sharing which disappeared when schools had to 'compete' with each other in league tables.***
>
> **(J.Edwards)**

The NCETM will share all the underpinning evidence in the report with partners and stakeholders and will explore in greater depth the messages in the first three themes and use them to inform and influence its work.

References

Ahmed, A. (1987). Better Mathematics: A Curriculum Development Study. London: HMSO.

Cockcroft, W. H. (1982). Mathematics Counts. London: HMSO.

DfES. (2005). Improving Learning in Mathematics. London: Standards Unit, Teaching and Learning Division.

QCA. (2007). Mathematics: Programmes of Study for Key Stage 3 & 4 and attainment targets.

Smith, A. (2004). Making Mathematics Count: The Stationery Office Ltd.

Swan, M. (2006). Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices. London: National Institute for Advanced and Continuing Education (NIACE); National Research and Development Centre for Adult Literacy and Numeracy (NRDC).

Appendix 1
Questionnaire used in colloquia and for online responses

Prompts for Session 1a **Practice**

You are asked to write a brief report on a memorable and successful lesson

Age/ability range
What was the mathematical task?
How was the session/task introduced?
How was the session/task sustained?
How was the session/task concluded?
What were the critical moments?
What mathematics was learnt? (on plan and off plan) and what is the evidence of learning?
How was that mathematics learnt?
Other memorable outcomes
Reference to any resources used

Prompts for Session 1b

Principles

Ideal and implemented values

Write "**A**" in the appropriate box on each row to show your vision for an ideal mathematics curriculum.
Write "**B**" in the appropriate box on each row to show the values implied by the curriculum that is currently implemented in most schools and other institutions.

4 = almost all mathematics lessons should contain this aspect
3 = most mathematics lessons should contain this aspect
2 = less than half of mathematics lessons should contain this aspect
1 = few mathematics lessons should contain this aspect

	More often ← → Less often			
Types of outcome and types of activity	**4**	**3**	**2**	**1**
Fluency in recalling facts and performing skills *For example:* Memorising names and notations Practising routine procedures				
Interpretations for concepts and representations *For example:* Discriminating between examples/non-examples Generating representations Constructing relationships Translating between representations				
Strategies for investigation and problem solving *For example:* Formulating questions/problems Developing/comparing strategies for solution Monitoring progress Interpreting/evaluating solutions Communicating results				
Awareness of the nature and values of the educational system *For example:* Recognising the purposes of learning maths Developing learning/reviewing strategies Knowing what others value				
Appreciation of the power of mathematics in society *For example:* Appreciate history/cultural foundations Creating/critiquing models of real situations Recognising uses/abuses of maths in society Gaining power over problems in ones own life				

Please write comments and notes on the back of this sheet.

Principles for teaching and learning

Here are some principles that have been emphasised in the recent "Improving Learning in Mathematics" materials.

Principles used in the Standards Unit material: "Improving Learning in Mathematics"

Teaching is more effective when it…	
• **builds on the knowledge learners already have**	This means developing formative assessment techniques and adapting our teaching to accommodate individual learning needs.
• **exposes and discusses common misconceptions**	Learning activities should expose current thinking, create 'tensions' by confronting learners with inconsistencies, and allow opportunities for resolution through discussion.
• **uses higher-order questions**	Questioning is more effective when it promotes explanation, application and synthesis rather than mere recall.
• **uses cooperative small group work**	Activities are more effective when they encourage critical, constructive discussion, rather than argumentation or uncritical acceptance. Shared goals and group accountability are important.
• **encourages reasoning rather than 'answer getting'**	Often, learners are more concerned with what they have 'done' than with what they have learned. It is better to aim for depth than for superficial 'coverage'.
• **uses rich, collaborative tasks**	The tasks we use should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, encourage 'what if' and 'what if not' questions.
• **creates connections between topics**	Learners often find it difficult to generalise and transfer their learning to other topics and contexts. Related concepts (such as division, fraction and ratio) remain unconnected. Effective teachers build bridges between ideas.
• **uses technology in appropriate ways**	Computers and interactive whiteboards allow us to present concepts in visual dynamic and exciting ways that motivate learners.

How far do you agree with these?
What principles would you add?
Are there principles, commonly believed by teachers that are positively unhelpful for learning mathematics? If so, then what are they?

Now share and discuss your ideas and their implications.

Prompts for Session 1c

Linking Practice to Principles

In groups of 3 with members A, B & C

- A spends about 5 minutes describing the lesson reported in Session 1a.
- B & C listen and comment and question in the light of the values and principles discussed in Session 1b.
- B may take notes.
- A, B and C identify which values were being exemplified in the presented example.
- This is recorded on the sheet below and attached to the example.
- A becomes B, B becomes C and C becomes A, and the process is repeated twice.

In the light of the discussion participants may revise their lesson/task report or select an alternative.

Response Sheet for Session 1c

Linking values and Principles to Practice

My lesson/task report illustrates the following value(s) and principle(s) in practice:
(please tick the appropriate box(es) in each table)

VALUES

Fluency in recalling facts and performing skills	
Interpretations for concepts and representations	
Strategies for investigation and problem solving	
Awareness of the nature and values of the educational system	
Appreciation of the power of mathematics in society	

How does the account exemplify this/these value(s)?

PRINCIPLES

Teaching is more effective when it…

builds on the knowledge learners already have
exposes and discusses common misconceptions
uses higher-order questions
uses cooperative small group work
encourages reasoning rather than 'answer getting'
uses rich, collaborative tasks
creates connections between topics
uses technology in appropriate ways

How does the account exemplify this/these principle(s)?

Prompts for Session 2

Identifying & Overcoming Obstacles

What are the main factors that inhibit or modify practice so that it becomes less than effective?

(a) What are the major obstacles to progress?
How do these obstacles function?

(b What practical steps can we take to help ourselves and others to overcome these obstacles?

Prompts for Session 3

A Personal response

Please write here any comments you have on the nature and structure of today's colloquium.
Please write here how you might continue and develop further some of the work started today – individually and/or back at your place of work.
Please write here how you might continue and develop further some of the work started today – with other colleagues in the region who have attended this colloquium.
Please write here how you might continue and develop further some of the work started today – with other colleagues nationally by using the Centre's portal.

APPENDIX 2

List of participants

Initial London Conference

Afzal Ahmed	The Mathematics Centre, University of Chichester
Patricia Alexander	Goldsmiths College – University of London
Barbara Allen	Centre for Mathematics Education, The Open University
Bob Ansell	University of Northampton
Jenni Back	Middlesex University
David Baker	Institute of Education, University of London
Chris Belsom	Ampleforth College
Carolyn Brooks	Anglia Ruskin University
Lynn Churchman	National Mathematics Partnership
Alison Clark-Wilson	University of Chichester
Julia Croft	University of Bedfordshire
Kath Cross	Retired HMI
John Dabell	Consultant
Els De Geest	University of Oxford
Pat Drake	University of Sussex
Stella Dudzic	Mathematics in Education and Industry
Julie-Ann Edwards	University of Southampton
Steve Feller	Edge Hill University
Tony Gardiner	University of Birmingham
Joy Garvey	Royal Borough of Kingston on Thames
Karen Gladwin	Anglia Ruskin University
Jenny Golding	Mathematical Association
Richard Goodman	University of Brighton
Jeffrey Goodwin	Edexcel
Rose Griffiths	University of Leicester School of Education
Linda Haggarty	The Open University
Matt Homer	School of Education, University of Leeds
Ros Hyde	University of Southampton
Fahrat Ibrahim	St Martin's College
Holly Isherwood	The Royal Institution
Jane Imrie	The NCETM
Keith Jones	University of Southampton
Jane Jones	Specialist Adviser for Mathematics
Vinay Kathotia	Clothworkers' Fellow in Mathematics – The Royal Institution of Great Britain
Lisa Kirby	CEA@Islington
Peter Lacey	ECARDA Ltd.
Michael Ling	The Royal Statistical Society
Tony Mann	University of Greenwich
Sarah Mann	ECARDA Ltd.
David Martin	NANAMIC
John Mason	The Open University
Colin Matthews	The NCETM
Dave Miller	AMET (Association of Mathematics Education Teachers) & Keele University
Debbie Morgan	University of Northampton
Bernard Murphy	MEI
Andrew Noyes	University of Nottingham
Adrian Oldknow	The Mathematical Association
Mark Patmore	Nottingham Trent University
Jennie Pennant	BEAM Education
Sue Pope	Qualifications and Curriculum Authority
Roger Porkess	Mathematics in Education and Industry
Dave Pratt	Institute of Education, University of London
David Reynolds	University of Plymouth
John Rickwood	Burlington Junior School
Dan Robinson	Specialist Schools and Academies Trust
Sue Sanders	University of Swansea
Malcolm Swan	University of Nottingham
Ruth Swinton	Mathematical Association
Ruth Tanner	Lodge Park Technology College
Howard Tanner	Swansea School of Education
Andrew Taylor	AQA
Lin Taylor	St Martin's College
Sidney Tyrrell	Coventry University
Geoff Wake	University of Manchester
Linton Waters	Secondary National Strategy for school improvement
Anne Watson	University of Oxford
David Wright	Newcastle University

Colloquia

1. Leicester

Barry Ablitt	Bishop Grosseteste University College Lincoln
Doreen Connor	Nottingham Trent University
Alan Cossins	Vocational Mathematics Project
Nikki Cruickshank	Bridgewater School
Liz Durham	Southend Adult Community College
Michael Elliott	St Paul's Catholic School
Peter Hough	The NCETM
Helen Morrell	Kimbolton School
Ray Sutton	The NCETM
Maggie Thomas	Northampton College
Ray Twigg	HMP Birmingham (Derby College)
Frances Watson	Gamlingay Village College
Jan Watson	School of Education, University of Leicester

2. Bath

Joan Ashley	The NCETM
Steffi Bludovsky	Colyton Grammar School
Sue Briggs	The Castle School,Taunton
Pete Griffin	The NCETM
Chris Haynes	DCS
Norma Honey	The NCETM
James Knightbridge	Blandford School, Dorset
Keith Noble	Poole Grammar School
Lee Northern	Cornwall's Children's Services Authority
Richard Perring	DCS
Jim Thorpe	The NCETM

3. Canterbury

Rosa Archer	St Mary's University College
Fiona Chapman	The Folkestone School for Girls
John Dore	University of Kent
Graham Griffiths	LLU/ London South Bank University
Sue Houston	Hastings College
Andrea Kite	Archers Court Maths and Computing College
Elaine Lambert	Orpington College
Snezana Lawrence	St Edmund's Catholic School
Pauline Noble	Hartsdown Technology College
Joan Ostheimer	Mid Kent College
Cheryl Periton	The NCETM
Margaret Sangster	Canterbury Christ Church University
Paula Stone	Canterbury Christ Church University
Bob Vertes	St Mary University

4. Newcastle

Viv Brown	The NCETM
Margaret Colley	Middlesbrough LA
Chris Cook	Heaton Manor School
Shirley Fall	Durham County Council
Alasdair Findlayson	Northumberland County Council
Tony Harries	Durham University
Steve Humble	The NCETM
Julia Gibb	Darlington LA
Brian Hill	Durham LA
Wendy Truscott	North Tyneside Council
Barbara Thompson	North Tyneside Council

5. Manchester

Liz Bell	Sherburn High School
Shirley Bertenshaw	Bolton Community College
Ian Brazewell	Manchester Adult Education Service
Caroline Cairns	Trafford College
Rachel Coombes	Craven College, Skipton
Nigel Culverwell	Manchester Adult Education Service
Dan Curran	Norton College
Chris Dallow	Worcester Sixth Form College
Richard Deakin	Failsworth School
Sue Evans	Lakes College
Damian Griffith	Failsworth School
Sue Harkness	Liverpool LA
Liz Henning	The NCETM
Teresa Kent	Wilberforce College
Rob Kinnersley	Shrewsbury Sixth Form College
Anna Kirwan	Stockport College
Emma Lawrenson	Myerscough Skills Tutor
Barbara Masters	The Sheffield College
Ken McKelvie	University of Liverpool
Janet Pass	Stockport Continuing College
Hannah Pomroy	Skipton Girls High School
Ian Porteus	University of Liverpool
Rob Simpson	South Hunsley School
Susan Wall	The NCETM
Pam Wignall	West Cheshire College
Pauline Williams	Ballakermeen High School
Will Wilson	Salford LA

6. Peterborough

John Bean	PFEG
Alison Brunt	The NCETM
Marion Dovey	Selby College
Christine Hines	Leeds Metropolitan University
Lynne McClure	Primary Project Box
Julie Pattison	North Yorkshire County Council
Pamela Wylie	Consultant

NOTES

www.ncetm.org.uk

A Department for Children, Schools and Families initiative to enhance professional development across mathematics teaching.