Resources to support the pilot of functional skills

Teaching and learning functional mathematics
Teaching and learning functional mathematics
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Preface

This resource has been prepared by the Functional Skills Support Programme for use in the centres that will be piloting functional skills. It has been updated from the publication ‘Teaching and learning functional mathematics’ that was produced in 2007.

In addition to this publication, FSSP has revised the following related publications:

1. Managing delivery of functional skills
2. Teaching and learning functional English
3. Teaching and learning functional Information and Communication Technology (ICT).

The aim of these four publications is to enable centres to move forward with the implementation of the pilot for functional skills. It is expected that most specialist teaching staff and leaders/managers in pilot centres will also have attended CPD sessions, launch events, or in-house training organised by the Functional Skills Support Programme.

Clearly, different practitioners, coming from different backgrounds, have very different areas of familiarity, interest and concern in relation to functional skills. An important aim of this material is to encourage common levels of understanding, so that teachers and leaders/managers coming from school, college, training provider, prison education and so on can develop a shared understanding, vocabulary and approach to functional skills that, while fit for each setting, have a common core. It is also important to understand that this publication is about teaching and learning. It does not give guidance on preparing learners for summative assessment.
Introduction to functional skills

What are functional skills?

Functional skills are essential skills in English, mathematics and ICT that enable everyone to deal with the practical problems and challenges of life – at home, in education and at work. They are essential to all our lives. For example, they help us recognise good value deals when making purchases, in writing an effective application letter, or when using the internet to access local services or online banking. They are about using English, mathematics and ICT in everyday situations.

Functional skills are a key to success. They open doors to learning, to life and to work. These skills are valued by employers and further education and are a platform on which to build other employability skills. Better functional skills can mean a better future – as learners or as employees.

Functional skills are not an ‘add on’. Although assessed separately, they will be an essential part of the secondary curriculum, embedded in the programmes of study for English, mathematics and ICT at Key Stage 3 and Key Stage 4, and a component of GCSEs, Diplomas, Foundation Learning Tier (FLT) and Apprenticeships. They will also be available as stand-alone qualifications for young people and adults. Functional skills are based on a problem-solving approach and will be developed in a practical way through discussion, thinking and explanation, right across the curriculum.

It is therefore important to recognise and promote that functional skills are essential for:

- getting the most from education and training
- the personal development of all young people and adults
- independence – enabling learners to manage in a variety of situations
- developing employability skills
- giving people a sound basis for further learning.

The implications for teaching and learning are significant and will need to be introduced gradually and thoughtfully, but they do not threaten aspects of existing good practice. Helping learners to become more ‘functional’ is supported by existing practices including:

- a focus on applied learning
- learner-centred approaches
- active learning and a problem-centred approach
- partnership learning
- assessment for learning.
How are functional skills being developed?

The standards

QCA has developed draft standards for functional English, mathematics and ICT at Entry levels 1, 2 and 3, Level 1 and Level 2 (QCA 2007). Figure 1 shows how these levels relate to the Qualifications and Credit Framework.

Figure 1

<table>
<thead>
<tr>
<th>Functional skills levels</th>
<th>Qualifications and Credit Framework</th>
<th>Examples of qualifications at each level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry 1</td>
<td>Entry</td>
<td>▪ Adult Literacy and Numeracy certificates</td>
</tr>
<tr>
<td>Entry 2</td>
<td>1</td>
<td>▪ GCSEs grades D-G</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 1 Key Skills</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 1 Certificates in Adult Literacy and Numeracy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 1 NVQ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Foundation Diploma</td>
</tr>
<tr>
<td>Entry 3</td>
<td>2</td>
<td>▪ GCSEs grades A*-C</td>
</tr>
<tr>
<td>Level 1</td>
<td>1</td>
<td>▪ Level 2 Key Skills</td>
</tr>
<tr>
<td>Level 2</td>
<td>2</td>
<td>▪ Level 2 Certificates in Adult Literacy and Numeracy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 2 NVQ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ BTEC First</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Advanced Diploma</td>
</tr>
<tr>
<td>Level 3 (NB standards not yet drafted)</td>
<td>3</td>
<td>▪ AS and A levels</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 3 Key Skills</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Level 3 NVQ</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ BTEC National</td>
</tr>
<tr>
<td></td>
<td></td>
<td>▪ Advanced Diploma</td>
</tr>
</tbody>
</table>
A learner who is ‘functional’ with mathematics, English and/or ICT is able to:

- consider a problem or task
- identify the functional mathematics, English and/or ICT skills that will help them to tackle it
- select from the range of skills in which they are competent (or know what help they need and who to ask)
- apply them appropriately.

This interplay of the four factors means, for example, that tackling a complex problem in a situation with which a learner is unfamiliar but that requires relatively undemanding English/mathematics/ICT skills may involve a higher level of ‘functionality’ than a relatively straightforward task in a familiar context that requires more advanced ‘subject’ skills. It is the combination of the four factors that confirms the functional skill level.

**A problem solving approach**

Functional skills are about identifying problems or challenges, selecting from the knowledge that we have, or knowing where to get it, and applying that knowledge to find effective solutions.

A key characteristic of functional skills is that they are based on a problem solving approach. Learners who are ‘functionally skilled’ are able to use and apply the English/mathematics/ICT they know to tackle problems that arise in their life and work.

Clearly, teachers cannot know what English/mathematics/ICT their learners will use as they move through their lives. This means that we cannot identify a curriculum core that every learner will use. Instead, and much more powerfully, learners should be taught to use and apply the English/mathematics/ICT that they know, and to ask for help with the areas in which they are less confident.
Teaching and learning functional mathematics: Introduction to functional skills

Why are functional skills needed?

Functional skills are needed for people to thrive. Functional skills are important in achieving the outcomes of the Government's Green Paper ‘Every Child Matters’ (DfES 2003), particularly:

- enjoy and achieve
- make a positive contribution
- achieve economic well-being.

The new qualifications have the potential to be an inspiring teaching, training and learning experience, which could improve chances for learners.

Functional skills have an impact on our adult lives too: the National Research and Development Centre (NRDC) has shown that people with poor literacy and numeracy have worse physical and mental health and low self-esteem, live in a poorer standard of accommodation, have more family breakdowns and are more likely to have been in trouble with the police. ICT skills are also increasingly important – they unlock information and help us communicate locally, nationally and worldwide.

**Functional skills are needed to access education and training**

The ‘Gilbert Review’ (2020 Vision, DfES, 2006) said that, without functional skills – being able to use English, mathematics and ICT as a matter of course whenever they are needed – pupils would find it ‘almost impossible to succeed’ because of the difficulty they would have in accessing the secondary curriculum. Functional skills will contribute to achievement of schools’ targets.

<table>
<thead>
<tr>
<th>Achievement and Attainment Tables</th>
<th>The AAT points for functional skills qualifications achieved in schools and colleges are:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level 2 = 23 points</td>
</tr>
<tr>
<td></td>
<td>Level 1 = 12.5 points</td>
</tr>
<tr>
<td></td>
<td>Entry 3 = 7 points</td>
</tr>
<tr>
<td></td>
<td>Entry 2 = 6 points</td>
</tr>
<tr>
<td></td>
<td>Entry 1 = 5 points</td>
</tr>
<tr>
<td></td>
<td>The points for Levels 1 and 2 are in addition to points allocated for other qualifications such as GCSEs and Diplomas.</td>
</tr>
</tbody>
</table>
The ability to apply functional skills is also crucial to accessing further and higher education. Universities and colleges have reported that weak functional skills have a negative impact on the number of students who complete a degree.

**Functional skills are needed for national prosperity**
Achieving functional skills qualifications will help poorly-qualified adults in the economic marketplace, enabling them to earn a living and contribute to national prosperity.

People who are more highly qualified are more likely to be employed and to earn more. The ‘Leitch Report’ (2006) found that, although school standards have improved and more young people than ever are achieving five good GCSEs, ‘…more than one in six young people leave school unable to read, write and add up properly…’. The Review emphasises the critical importance of improving functional literacy and numeracy.

**Functional skills are needed for employability**
Literacy, numeracy, team working and communication are relevant in most jobs and the Leitch Report set targets of:
- 95% of adults to achieve functional literacy and numeracy
- more than 90% of adults to be qualified to at least Level 2 by 2020.

The CBI found that:

‘Weak functional skills are associated with higher unemployment, lower earnings, poorer chances of career progression and social exclusion… The time has come to ensure that school-leavers in future have the functional skills they need for work and daily life. In short, British business sees concerted action on functional skills as a key priority.’

(Working on the Three Rs, CBI, 2006)

Functional skills will help to ensure that employers can recruit workers with the skills they need. They will be a badge of competence, showing that potential recruits can cope with the demands of the workplace, offering a single ladder of achievement and progression with each level incorporating and building on the level/s below.

**Assessment**
Standards are, of course, only the first stage in developing qualifications. When they are finalised, QCA works with the awarding bodies to develop the assessment methods and the qualifications.

The assessment methods for functional skills qualifications must be fit for purpose across a wide range of learners in a wide range of contexts. It may be that no one method will be appropriate to all settings.
During the pilot, 11 awarding bodies are piloting a range of models of assessment. QCA has produced three documents, one for each functional skills subject, entitled ‘Assessment arrangements and principles for pilot’. Many of the principles are common to all three functional skills, including:

- the assessment can be entirely task-based, or a combination of tasks with test-style items
- the assessment should not be entirely test-based
- assessment items may be externally set by an awarding body or requirements may be externally set and provide for internally contextualised task-based assessments
- assessment is of the candidate’s own ability to solve a problem or reach an outcome by independent application of skills.

For details of assessment, you should contact your awarding body.

**The pilot**

What has been learned from the trials and from the pilot will continue to inform future decisions. Over 2000 centres are now involved, most of whom are schools, although they also include colleges, training providers, work-based provision, adult and community settings and secure settings.

**Timelines**

<table>
<thead>
<tr>
<th>Start date</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 2007</td>
<td>Three-year pilot (approximately 1000 centres in the first year) of functional English, mathematics and ICT in a range of contexts, including stand-alone.</td>
</tr>
<tr>
<td>September 2008</td>
<td>All three functional skills piloted within the first tranche of Diplomas (construction and the built environment, creative and media, engineering, society health and development, IT).</td>
</tr>
<tr>
<td>September 2010</td>
<td>Functional English, mathematics and ICT available nationally.</td>
</tr>
</tbody>
</table>
References


Useful sources of information

**Functional Skills Support Programme**
Go to functionalskills@lsneducation.org.uk or telephone the Helpline on 0870 872 8081
www.standards.dcsf.gov.uk/nationalstrategies

**DCSF 14-19 website** at www.dcsf.gov.uk/14-19 – go to ‘Qualifications’ and then ‘Functional skills’.

**QCA website** at www.qca.org.uk/qca_6062.aspx has information about the functional skills standards and the pilot.

**The QIA Excellence Gateway** at http://excellence.qia.org.uk has links to downloadable versions of teaching and learning resources to support the delivery of functional skills.

Many of the **awarding bodies’** websites have sections dedicated to functional skills.
Teaching and learning functional mathematics

Overview

‘Teaching and learning functional mathematics’ is intended to support teachers of mathematics as they prepare courses that lead to qualifications that include functional mathematics. There are six sections.

The first section, the Introduction, sets out what functional mathematics is, what is expected to change as a result of the Government’s vision for functional mathematics, and how teachers should use the Qualifications and Curriculum Authority (QCA) document: Functional skills standards: mathematics (QCA/07/3472).

The second section, ‘The problem solving process’, describes how to introduce the problem solving process into mathematics lessons. It provides users with guidance on how to teach through problems so that learners can become more functional with their mathematics. It includes many examples of suitable problems.

‘Writing your own contextualised activities’, the third section, gives strategies for building your own activities to support learners in becoming more functional with using their mathematics. It explains and exemplifies how to devise and use activities that are based in real-world contexts. This will help teachers to promote motivation in their learners by increasing the relevance and authenticity of the tasks they devise.

The fourth section, ‘Cross-curricular activities’, gives examples of a range of suitable activities that are set in a wide range of contexts and can be used or adapted to support the problem solving process.

This is followed by a section called ‘Assessment, progression and mastery’ that sets out how teachers should assess their learners’ progress in functional mathematics. This section considers how to assess process skills and how to use the QCA standards to support work in this area.

The final section, ‘Resources’ lists a large number of relevant resources and sources of information.

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The school defines mastery of functional skills as ‘the ability to select and apply the appropriate skill to a novel situation’. In order to develop functional mathematics in Diplomas, the Principal Learning teachers will supply the novel and applied contexts, while the mathematics team will map where opportunities for the application of mathematics appear in the programmes of work.

The centre is working towards creating a shared terminology for the skills and processes across all learning, and aims to move to a skills-based curriculum for all learners. The ability of learners to apply mathematics as part of the generic learning outcomes in the Diploma is central to this ambition.
1. Introduction

Contents
1.1 What is functional mathematics?
1.2 The vision for functional mathematics
1.3 Teaching functional mathematics
1.4 How to read the standards
1.5 Level differentiation

1.1 What is functional mathematics?

The DCSF’s generic definition of functional skills notes that functional skills will:

‘provide an individual with the essential knowledge, skills and understanding that will enable them to operate confidently, effectively and independently in life and at work. Individuals of whatever age who possess these skills will be able to participate and progress in education, training and employment as well as develop and secure the broader range of aptitudes, attitudes and behaviours that will enable them to make a positive contribution to the communities in which they live and work.’

The vision described is of learners:

- developing the practical applied skills needed for success in work, learning and life
- tackling the skills gap, improving productivity, enterprise and competitiveness
- becoming more confident in their studies in further and higher education
- becoming more confident in interaction with people in their lives.

Functional mathematics will contribute to this agenda. Learners who are functional with mathematics are able to use and apply the mathematics they know to address problems that arise in their life and work.

1.2 The vision for functional mathematics

The introduction to Functional skills standards: mathematics states that:

‘The term “functional” should be considered in the broad sense of providing learners with the skills and abilities they need to take an active and responsible role in their communities, everyday life, the workplace and educational settings. Functional mathematics requires
Teaching and learning functional mathematics: 1. Introduction

learners to use mathematics in ways that make them effective and involved as citizens, to operate confidently in life and to work in a wide range of contexts.

The mathematics standards are essentially concerned with developing and recognising the ability of learners to apply and transfer skills in ways that are appropriate to their situation...

For mathematics to be useful, learners must have the skills and confidence to apply, combine and adapt their mathematical knowledge to new situations in their life and work.

It is important to recognise that all mathematics can be used in these ways, and that teachers cannot know what mathematics their learners will use as they move through their lives. This means that we cannot identify a curriculum core that every learner will use. Instead, and much more powerfully, learners should be taught to use and apply the mathematics that they know and have learned, and to recognise when they need to develop additional skills.

It is essential to think of learners becoming functional with their mathematics, rather than thinking there is a vital body of mathematical material, known as functional mathematics.

1.3 Teaching functional mathematics

For teachers, helping learners to become functional with mathematics means helping them to:

- recognise situations in which mathematics can be used
- make sense of these situations
- describe the situations using mathematics
- analyse the mathematics, obtaining results and solutions
- interpret the mathematical outcomes in terms of the situation
- communicate results and conclusions.

This will mean that learners should experience sessions that have a significantly new emphasis and focus on problems of sufficient scope to permit these processes to flourish. Learners need to demonstrate the ability to use and apply straightforward mathematical skills in complex contexts. This is different from much mathematics teaching in which learners often use challenging mathematics in very simple contexts, or entirely out of context.

The problems that learners meet in sessions with this new emphasis may sometimes be complicated and extensive. If this is the case, the problems will need to be solvable using mathematics that the learners have met before.
It is important that learners are not told, at the time a problem is set, which of the mathematical tools they have at their disposal will actually be needed. Selecting the right tools is a core aspect of becoming functional with mathematics.

The problems should also be plainly relevant to learners, appealing to them by being motivating, interesting and realistic. Mathematics teaching should reveal how mathematics is used in life, enabling learners to gain experience of the breadth of applications of the subject. It is important for specialist mathematics teachers to liaise with colleagues to identify and maximise the opportunities to embed functional mathematics in other curriculum areas.

Part of the push towards relevance and motivation depends on making the use of ICT integral to teaching and learning mathematics. When encouraging learners to become functional with mathematics, ICT should be given an important role that reflects its significance in life and in the workplace as well as its potential to enhance and motivate mathematics learning. Indeed, it is good practice to give learners opportunities to use all three functional skills when tackling problems, as is often the case in real life.

The implications for teaching and learning of the features of functional mathematics described above are significant. They will need to be introduced gradually and thoughtfully but they do not threaten aspects of existing good practice. ‘Teaching and learning functional mathematics’ sets out some of the ways in which making adjustments to help learners become more functional with mathematics is supported by existing practices including:

- learning through application
- learner-centred approaches
- active learning and a problem-centred approach
- partnership learning
- assessment for learning.

1.4 How to read the standards

Functional mathematics standards have been issued for the first three levels of the National Qualifications Framework – Entry level, Level 1 and Level 2. As usual, Entry level is subdivided into Entry 1, Entry 2 and Entry 3 to reflect the importance of small incremental steps in learning for learners at these levels. For ease of reference, Entry 1 is broadly comparable in demand with National Curriculum level 1, Entry 2 with National Curriculum level 2 and Entry 3 with National Curriculum level 3. Level 1 is comparable with GCSE grades D-G and Level 2 is comparable with GCSE grades A*-C.

The standards are set out in a single document, published by QCA. After a brief introduction, the document sets out the standards in two sections. The
first and most important section sets out the underpinning process skills that make it clear what learners have to do to demonstrate that they are functional in mathematics. The second section indicates ways in which performances at the various levels can be differentiated.

The process skills are fundamental to the standards for mathematics, and do not change as learners progress through the levels. They are the principal learning targets for functional mathematics and are set out in three columns – Representing, Analysing, Interpreting – each with a number of bulleted statements. (See Figure 1.1, page 19.)

These learning targets are not to be interpreted as three distinct areas of study, nor are the bulleted statements separate statements of attainment, to be ticked off as each is achieved. On the contrary, the three column headings and the individual bulleted statements in the process skills must be interpreted as describing aspects of a single larger process. That process, which is akin to problem solving, is the process of being functional with mathematics.

It is helpful to think about how far the mathematics lessons you currently teach or observe focus exclusively on the analysing aspect of the process skills. This can easily happen when there is pressure to teach many mathematical techniques in a limited time. In functional mathematics, it is very important for learners to experience the need to decide for themselves whether a problem can be addressed using mathematics, what mathematics might help, and how the problem should be set out mathematically (represented). An example of such a problem is to find a way to help someone who lives in a town to visualise the size of an acre or a hectare.

It is also important for learners that they are asked what the mathematical solution means in terms of the initial situation. This is what is meant by ‘interpreting the solution’. For example, the calculation $30 ÷ 4 = 7.5$ is interpreted quite differently when deciding how many four-seater cars are needed to transport 30 people from the way it is interpreted when deciding how many sweets each of four people can have from a bag of 30 sweets, shared equally. Employees in the workplace will frequently be required to provide a mathematically clear account of how a solution was found and interpreted.
### Figure 1.1 The process skills

<table>
<thead>
<tr>
<th>Representing</th>
<th>Analysing</th>
<th>Interpreting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making sense of situations and representing them</td>
<td>Processing and using mathematics</td>
<td>Interpreting and communicating the results of the analysis</td>
</tr>
</tbody>
</table>

**A learner can:**
- recognise that a situation has aspects that can be represented using mathematics
- make an initial model of a situation using suitable forms of representation
- decide on the methods, operations and tools, including ICT, to use in a situation
- select the mathematical information to use.

**A learner can:**
- use appropriate mathematical procedures
- examine patterns and relationships
- change values and assumptions or adjust relationships to see the effects on answers in the model
- find results and solutions.

**A learner can:**
- interpret results and solutions
- draw conclusions in the light of the situation
- consider the appropriateness and accuracy of the results and conclusions
- choose appropriate language and forms of presentation to communicate results and conclusions.

### 1.5 Level differentiation

The second section of the standards (‘Level differentiation’) describes performance at the different levels. It begins by listing ways in which problems that can be solved using mathematics can differ in their demands on learners. The four features identified are:

- the complexity of the situation or problem
- the familiarity to the learner of the situation or problem
- the technical demand of the mathematics required
- the independence of the learner in tackling the situation or problem.

These four features of problems could be regarded as dimensions of difficulty. Plainly, a more complex situation will be more challenging to understand and represent mathematically. Conversely, a learner given extra support will find a problem more approachable. In many mathematics lessons the focus is on mathematical techniques, and the difficulty of the techniques determines the difficulty of the work. In functional mathematics, however, it is important to recognise that many real-world problems are complex and unfamiliar. Dealing with such problems could become too challenging if the technical demand of the mathematics were also at the limit of the learner’s capability.
In this second section of the standards, there is a page for each level. These pages give information about performance and coverage/range, and contribute to describing performance at a particular level. Figure 1.2 shows this for Level 1.

**Figure 1.2**

**Mathematics: level 1**

The standard at level 1 is underpinned by the process skills of representing (making sense of situations and representing them), analysing (processing and using the mathematics) and interpreting (interpreting and communicating the results of analysis).

<table>
<thead>
<tr>
<th>Performance</th>
<th>Coverage and range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Learners can:</td>
<td>Content and skills are equivalent to national curriculum mathematics levels 1–4, the adult numeracy standards and the application of number key skill, level 1</td>
</tr>
<tr>
<td>– understand practical problems in familiar and unfamiliar contexts and situations, some of which are non-routine</td>
<td>Learners can:</td>
</tr>
<tr>
<td>– identify and obtain necessary information to tackle the problem</td>
<td>– understand and use whole numbers and recognise negative numbers in practical contexts</td>
</tr>
<tr>
<td>– select and apply mathematics in an organised way to find solutions to practical problems for different purposes</td>
<td>– add, subtract, multiply and divide whole numbers using a range of mental methods</td>
</tr>
<tr>
<td>– use appropriate checking procedures at each stage</td>
<td>– multiply and divide whole numbers by 10 and 100 using mental arithmetic</td>
</tr>
<tr>
<td>– interpret and communicate solutions to practical problems drawing simple conclusions and giving explanations.</td>
<td>– understand and use equivalencies between common fractions, decimals and percentages</td>
</tr>
<tr>
<td></td>
<td>– add and subtract decimals up to two decimal places</td>
</tr>
<tr>
<td></td>
<td>– solve simple problems involving ratio, where one number is a multiple of the other</td>
</tr>
<tr>
<td></td>
<td>– use simple formulae expressed in words for one- or two-step operations</td>
</tr>
<tr>
<td></td>
<td>– solve problems requiring calculation, with common measure including money, time, length, weight, capacity and temperature</td>
</tr>
<tr>
<td></td>
<td>– convert units of measure in the same system</td>
</tr>
<tr>
<td></td>
<td>– work out areas, perimeters and volumes in practical situations</td>
</tr>
<tr>
<td></td>
<td>– construct models and draw shapes, measuring and drawing angles and identifying line symmetry</td>
</tr>
<tr>
<td></td>
<td>– extract and interpret information from tables, diagrams, charts and graphs</td>
</tr>
<tr>
<td></td>
<td>– collect and record discrete data and organise and represent information in different ways</td>
</tr>
<tr>
<td></td>
<td>– find mean and range</td>
</tr>
<tr>
<td></td>
<td>– use probability to show that some events are more likely to occur than others</td>
</tr>
<tr>
<td></td>
<td>– understand outcomes, check calculations and explain results.</td>
</tr>
</tbody>
</table>
Teaching and learning functional mathematics: 1. Introduction

The ‘Performance’ part gives an illustration of what teachers may expect of learners in relation to the issues of complexity, familiarity and learners’ independence at the level. The performance statements should be regarded as indicating one way among many possible ways in which the difficulty of a problem may be expressed at the relevant level. A different problem with greater complexity but involving less independence, for example, could be of equivalent difficulty.

The ‘Coverage/range’ part indicates the technical demand of the mathematical skills and techniques that are likely to be used by learners performing at that level. The curriculum that may be relevant is not set out in full but is indicated by the references to levels in the box below the heading. These state that the listed content and skills should be regarded as equivalent to particular levels of the national curriculum, the related adult numeracy standard, and the related application of number key skill standard (levels 1 and 2 only). The coverage/range statements indicate some of the mathematical skills and techniques that are likely to be used by learners performing at that level. They are not a complete list, so other skills and techniques from the overall curriculum are equally valid for learners to use in achieving the level. It is vitally important that teachers do not regard the coverage/range statements as the list of skills and techniques that learners must show they can use to achieve mastery at the level.

Rather, mastery of the level must be judged in relation to the process skills. Learners must have demonstrated a sufficient grasp of the whole process at any level to be regarded as working at that level. The performance and coverage/range statements can be used to give teachers confidence that the problems solved by learners were of sufficient demand to describe their performance as achieving a particular level. There is more information about this in section 5, ‘Assessment, progression and mastery’ (page 99).
Post-16 consortium

The centres in this consortium use diagnostic assessments to identify a learner’s abilities in applying mathematics. They then deliver sessions to develop the gaps identified and create opportunities to apply the skills in components of the Diploma. The diagnostic assessments provide a framework for producing individual learning plans and for monitoring the learner’s progress in acquiring specific mathematical skills.

Being able to demonstrate a mathematical skill in the abstract is different from being able to use it in a practical context. The hallmark of the consortium’s approach is that the learner is not given abstract exercises but is instead presented with practical tasks that involve using functional mathematics in the context of activities in Principal Learning. A bank of support materials is being developed that the Principal Learning teacher can use to help learners develop the process skills they need to carry out specific tasks in the Line of Learning. The integration of functional mathematics into Principal Learning is likely to be both appealing and motivating for Diploma learners.
2. The problem solving process

Contents

2.1 Why a problem solving approach is especially relevant to functional mathematics
2.2 Asking questions
2.3 Adapting questions to other contexts
2.4 Creating a story
2.5 Looking for the mathematics
2.6 Justifying decisions
2.7 Classifying, ordering and sorting
2.8 Analysing solutions
2.9 Language problems

This section is designed to help you develop your learners’ problem solving skills. The ideas are built around functional skills but they can be applied to any aspect of mathematics at any level. The more experience learners have of working in this way, the better they will become at reflecting and thinking critically about their work. They are also likely to improve their learning across the mathematics curriculum and to develop more positive attitudes to the subject.

Mathematical problems require decisions to be made about the mathematics needed and the strategies to be used. Problems come in all shapes and sizes from single-stage closed textbook questions to open-ended investigations. The context can be purely mathematical or can be taken from real life, including contexts from other subjects. All have their challenges but if learners only experience textbook-style questions that are based on one topic and a method that has recently been taught, they will struggle to tackle problems that require the process skills of functional mathematics.

To be able to tackle problems that are more open-ended, learners need to ask questions about the context of the problem, for example: What is this telling me? Would it make any difference if...? They should be able to sort and organise information, including deciding what is relevant and what is redundant by seeing the problem and solution as a whole rather than as a lot of small pieces. This will encourage learners to look for and spot patterns and relationships, and generalise from them where appropriate.

Learners have to realise that there is not necessarily one correct way of tackling a problem and not necessarily only one correct answer to a problem. Choices have to be made but they need to be justified.
This section includes a range of ideas and activities that will encourage learners to develop these skills. They are not written as stand-alone sessions but as ideas to incorporate into ‘normal’ teaching and learning. They can be adapted for different levels and different topics. Some examples have been included to illustrate how this can be achieved, although it has not been possible to include the whole range of possible contexts and settings. The activities start by moving standard textbook problems from a very closed context into a more open one and encouraging learners to ask questions. Learners consider a range of appropriate contexts in which mathematics may be used, thus helping them gain ownership over the mathematics.

The activities are most effective when learners are working in pairs or groups. This encourages discussion, thinking and explanation. Explaining to others focuses the learner’s attention on the various features of the problem that influence its difficulty. This in turn helps learners to explore the situation more fully. The problems can be solved collaboratively using large sheets of paper and coloured felt-tipped pens so that all learners in the group are encouraged to be involved in a creative way.

Note: For a detailed discussion of this approach to teaching mathematics, see ‘Improving learning in mathematics: challenges and strategies’. This is included in *Improving learning in mathematics* (the ‘Standards Unit box’) published by the DfES in 2005 and available from the National Centre for Excellence in the Teaching of Mathematics (www.ncetm.org.uk). For details, see section 6, ‘Paper-based materials’, page 119.

### 2.1 Why a problem solving approach is especially relevant to functional mathematics

As explained in the Introduction, helping learners to become functional with mathematics means helping them to:

- recognise situations in which mathematics can be used
- make sense of these situations
- describe the situations using mathematics
- analyse the mathematics, obtaining results and solutions
- interpret the mathematical outcomes in terms of the situation
- communicate results and conclusions.

Learners will need to experience lessons with an emphasis on activities that have sufficient scope to permit all these processes to flourish. A problem solving approach permits learners to develop all the process skills in *Functional skills standards: mathematics*, because these process skills are, essentially, problem solving skills. Preparing learners in functional mathematics means helping them to develop problem solving process skills.
For mathematics, this is likely to involve considerable change in the curriculum for many learners. This is because many teachers concentrate on ensuring that their learners have been introduced to all the curriculum content (thus addressing the skill that the standards call ‘analysing’) without the ‘representing’ and ‘interpreting’ skills, and many examinations such as GCSE mathematics reward such an approach by giving little credit for representing and interpreting.

However, in the world outside mathematics lessons, we rarely know at the time a problem is posed whether or how mathematics will help solve it and, if so, what mathematics is needed. To solve problems in the wider curriculum, in life and in the workplace it is often necessary to go through the processes outlined in the standards.

Being functional with mathematics requires learners to demonstrate that they can represent situations using mathematics and interpret mathematical results in terms of the original situation. These process skills are best developed by learning to deal with substantial problems.

‘Representing’ is about being able to describe a situation mathematically. Some problems are represented by very commonplace methods, such as addition. This kind of representation can become so natural that we no longer notice ourselves deciding to use it.

However, learners will need to become consciously aware of the mathematics they need to use to solve a problem.

In all the examples in this section, learners are required to think for themselves. This is an experience that will stand them in good stead when they come across problems in the wider curriculum, in life or in the workplace and decide that a mathematical approach is needed.

2.2 Asking questions

This approach takes standard textbook or practice questions and requires learners to think beyond the question and its answer – they have to focus on the context of the question and investigate that. This kind of activity is about making sense of situations and representing them, as well as processing and using mathematics.

After learners have answered the original question, they could be asked ‘What other questions could be asked about this situation?’ These ideas can be collated and discussed by the group before being answered. Alternatively, specific questions could be asked first and then followed by the further question.
Question 1
It takes 1.75 metres of denim to make a pair of jeans. Denim costs £3.50 per metre.
(a) How much will the material for the jeans cost?
(b) If the price of denim rises by 5%, how much will the material cost?

What other questions could you ask about this situation?
- If the denim can only be bought in an exact number of metres, how much extra will you pay for the denim you do not use?
- How many pairs would you need to make to ensure that there is no wastage of denim?
- What is inflation at the moment? What would happen if you used that figure instead of 5%?
- How much discount would you need to get the price back to where it was before the price increase?
- What would the discount need to be if the increase was 10%, or 20%?
- Can you generalise from these examples?
- The actual price of the jeans is double the cost of the denim because of trimmings, labour and profit. How much will the maker charge for the jeans?
- Would you ever choose to have jeans especially made for you? If so, why, and if not, why not?
- How do you decide where to buy your jeans?

Question 2
Jenny goes shopping and buys two CDs priced at £6.99 each and three T-shirts costing a total of £13.50. She took £40 with her into town.

How much change will she have from her purchases?

What other questions could you ask about this situation?
- How much was each T-shirt? How do you know? Give some examples of possible prices.
- Is she likely to come home with all her change?
- How did she get home?
- What else might she spend her money on?
• Estimate her other expenditure.
• What would you spend the money on if you took £40 into town on Saturday?

Question 3
The monthly charge for a mobile phone is £25. This includes 300 minutes of free calls. After that there is a charge of 5p per minute.
Calculate the cost of using the phone for 540 minutes in one month.

What other questions could you ask about this situation?
• What other information would you want to know about the charges for this phone before you decide to buy it?
• What difference is it likely to make to the bill if calls after 6.00 pm are only 3p per minute?
• What is the method of payment of your mobile phone? (If you have not got a phone, ask a friend about theirs.)
• Would you consider changing to the phone in the question? Explain your reason.
• Why do some people have ‘pay as you go’ but others have a monthly rental?

Question 4
Claire wants to record four programmes on a video tape that is three hours long. The lengths of the programmes that she wants to record are 30 minutes, 45 minutes, 50 minutes and 40 minutes.
How much time will she have left on her tape when she has recorded them?

What other questions could you ask about this situation?
• What do you think the programmes are?
• How much of the time do you think is adverts?
• If Claire started watching the video at 6.00 pm when will she finish watching?
• What time do you think she will finish if she uses fast-forward when the adverts are on?
• What programmes would you like to record this week? How long will they last altogether?
Similarly, if a problem involves a graph as the solution or as part of the problem, learners can ask questions about the graph.

**Question 5**

Andrew sets off on a cycle ride as shown on the graph above. What is his average speed for the whole journey?

What other questions could you ask about this situation?

- Where did he fall off his bike?
- What happened at the end? Why do you think he did not go home?
- When was he riding at his fastest?
- Why do you think he was going fast on this part of the journey?
- What could have been happening after 3½ hours of the ride?
- Describe the whole journey in words.
- How would the graph have been different if he had got a puncture somewhere and had to walk the rest of the way?
- How could you change the graph so that his average speed is 4 km h\(^{-1}\)? Or 6 km h\(^{-1}\)?
Teaching and learning functional mathematics: 2. The problem solving process

- Is the graph realistic? If not why not? What assumptions have been made? How could you make it more realistic?

Even ‘simple’ questions from an exercise can be developed into more complex problems.

**Question 6**
Calculate $20 + 16 \times 5$

Ask learners to create a story or scenario that this calculation might represent. Once the story has been created the questions that can be asked about the context are limitless. Any sort of calculation problem can be used in this way (eg $34 \times 1.05$, or $4500 \times 3 + 2510 \times 2$).

### 2.3 Adapting questions to other contexts

Question 1 (below) could be adapted to other contexts by changing the focus of the activity from buying material for a pair of jeans to, for example, ordering ingredients to make a recipe in a catering context, as shown in the following adaptation.

**Question 1 Catering**
To make 50 bread rolls takes 1.6 kg of strong white flour, which costs £1.95 per kilogram.
(a) How much will the flour cost?
(b) If the price of flour rises by 5%, how much will the flour cost?

The other questions can also be adapted to the new context, for example in the catering context, as follows.

- If the flour can only be bought in an exact number of kilograms, how much extra will you pay for the flour you do not use?
- How many bread rolls would you need to bake so that there is no wastage of flour?
- What is inflation at the moment? What would happen if you used that figure instead of 5%?
- How much discount would you need to get the price back to where it was before the price increase?
- What would the discount need to be if the increase was 10%, or 20%?
- Can you generalise from these figures?
• The actual price of the bread rolls is double the cost of the flour because of the yeast, butter and sugar needed and the baker’s profit. How much will the baker charge for the bread rolls?
• Does the type of bread roll make a difference to the cost? Are brown bread rolls more expensive than white bread rolls? If so, why might that be, and if not, why not?
• How do you decide where to buy your bread rolls?
• What other questions could be asked about this situation?

This approach to adapting questions for learners can be applied to many learning and work contexts. Question 2 (below) could be adapted to a social care context, as follows, where care workers in a residential home are often asked to shop for residents.

**Question 2  Social care**

Jenny goes shopping for Ada, one of the residents, who wants four Lucky Dips at £1 each, a magazine that costs £1.30, two pairs of support tights that cost £6.15 a pair and two boxes of tissues at £0.75 a box. She gives Jenny £20 to buy everything.

How much change will Jenny have to give back to Ada?

The further questions would need to be adapted, but could include asking learners to decide how much extra change Jenny would have if the tissues were on special offer at £1.35 for two boxes, or what she could do if the tights had gone up in price by 50p a pair.

This approach can be applied to any problem that has a context relevant to the learners, including problems from other subject areas. Learners could, for example, tackle a problem about population from geography or one about the laws of motion from physics. These problems can be extended in the same way by asking further questions and extending the context.

### 2.4 Creating a story

It can also be interesting to ask learners to create a story about a graph or chart that has no labels on the axes. A graph or chart that is to be used as part of a question in an exercise problem could first be shown with its labels removed. This will make learners examine the graph carefully and think about the significance of each of its parts.
For example:

Adapting to other contexts

Similar graphs and charts could be used in learners’ own contexts to challenge them to think about their possible meaning in that context.

For example, the chart might represent sales of certain items in a retail setting, or the types of treatments requested in a beauty therapy salon. Charts and graphs are regularly used in many other curriculum areas, for example history, geography, physical education and science. Learners could be asked to interpret the graph or chart in the context of one of their other curriculum subjects.

2.5 Looking for the mathematics

Learners sometimes struggle to identify what mathematics is needed to solve a particular problem. Ask them to identify the mathematics in a range of everyday events by suggesting questions that could be asked about them.
The questions do not have to be answered but creating them and discussing them will encourage learners to decide what mathematics is involved in particular situations. They may be surprised at how much mathematics they can find.

Encourage learners to ask as many different questions as they can and then, as a class or as a small group, write problems that require some or all of these questions to be answered.

Learners could also be asked to create a problem about each scenario that does not require any mathematics to answer it. Other learners can then try to identify some relevant mathematical questions that can be asked. Some of the problems may be trivial but it is all part of the process of sorting out whether or not a problem lends itself to mathematics. In this way, learners will get used to ‘looking for the mathematics’ so that, when they are faced with a problem to solve, they will be able to give a sensible decision as to whether it requires mathematics to solve it.

This can be extended by giving learners particular problems that need solving and asking them what, if any, mathematics they need to solve the problem. This is particularly appropriate if problems from other curricular areas are used. Learners can firstly identify all the mathematics that there is in the context of the problem, possibly by considering what questions can be asked, and then decide whether any of it is relevant to tackling that particular problem.

This will encourage learners to think about all the mathematics involved in any problem and whether or not it is appropriate to use mathematics to solve the problem.

The following are examples of scenarios that could be used.

<table>
<thead>
<tr>
<th>What mathematical questions could be asked about:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• a bus journey to work?</td>
</tr>
<tr>
<td>• making a cup of coffee?</td>
</tr>
<tr>
<td>• watching a DVD?</td>
</tr>
<tr>
<td>• going to watch a football match?</td>
</tr>
<tr>
<td>• booking a client for treatment, for example in a beauty salon?</td>
</tr>
<tr>
<td>• painting the skirting board of a room?</td>
</tr>
<tr>
<td>• planning a holiday?</td>
</tr>
<tr>
<td>• today’s weather?</td>
</tr>
<tr>
<td>• the London Marathon?</td>
</tr>
<tr>
<td>• the moon?</td>
</tr>
</tbody>
</table>
Alternatively, learners could be shown a visual image or information that is relevant to them and asked to ‘look for the mathematics’. The image or information could come from their vocational area, the local environment, or other curricular subjects, for example a map from geography, a graph from chemistry, a source from history, a design from technology or a formula from physics.

**Examples**

What mathematical questions could be asked about the information below?

Final results for 2007–08

<table>
<thead>
<tr>
<th>Hull City</th>
<th>Blackpool</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd in the Championship</td>
<td>19th in the Championship</td>
</tr>
<tr>
<td>Played 46</td>
<td>Played 46</td>
</tr>
<tr>
<td>Won 21; drawn 12; lost 13</td>
<td>Won 12; drawn 18; lost 16</td>
</tr>
<tr>
<td>Goals for 65; goals against 47</td>
<td>Goals for 59; goals against 64</td>
</tr>
<tr>
<td>Goal difference: +18</td>
<td>Goal difference: –5</td>
</tr>
<tr>
<td>Points: 75</td>
<td>Points: 54</td>
</tr>
</tbody>
</table>

What mathematical questions could be asked about this picture?
2.6 Justifying decisions

In problem solving there is often no such thing as one right answer, particularly for more open questions. It is the answer plus the justification that is important. If the justification is appropriate, sufficient and valid, then the answer is correct. Even if the question is closed, justification is needed. Learners sometimes find this a difficult idea to grasp but, once they learn to compare and discuss alternative solution strategies to problems, their confidence and flexibility in using mathematics increase. In activities of this kind, learners will be analysing mathematics and interpreting solutions, as required by the functional mathematics standards.

The ‘Agree or disagree’ activities suggested on pages 35–36 can be carried out as a class discussion or with pairs of learners sticking the statements onto large sheets of paper and adding their justification in writing. These activities or problems can be used to start a session or to review the learning at the end of a session. There is an opportunity here to encourage learners to express their reasoning clearly and logically using appropriate mathematics.

These activities or problems require learners to justify and give reasons for their decisions and so encourage their capacity to explain and convince. Initially the ‘Agree or disagree’ examples are closed, in the sense that the statement is either true or false. However, learners have to provide justification for their decision for each one; how they do that is up to them.

The examples given contain common errors and misconceptions and so are likely to provoke interesting and useful discussions which, it is hoped, will result in a decision that is clearly and carefully justified. The examples cover a range of mathematics but each one could be developed into a set about one particular aspect of mathematics. They could be replaced with examples containing statements from a range of vocational contexts that include common errors and misconceptions.

The ‘Statements to evaluate’ on page 37 are more provocative and learners have to give an opinion backed up with justification. They are intended to move learners on beyond the idea that there is always only one answer to a problem.

In ‘Sometimes, always, never’ on page 38 learners are given statements that they have to evaluate. They have to test the statements out using appropriate specific cases to decide which category each statement is in. As they go deeper into sorting out the problem, they may be able to generalise and explain their decision rather than only using examples and counter-examples.

In ‘Odd one out’ on pages 39–40 each card shows three similar mathematical objects. Each one of the three could be the odd one out. Learners have to justify each one being the odd one out in as many ways as they can.

Ideas that develop learners’ ability to explain, convince and prove can be devised at any level of difficulty and for any topic. They are particularly effective if they force learners to confront common difficulties and
misconceptions in a context that is relevant to them. All the above ideas could be replaced with examples from a vocational context or from other curricular areas. For example, the statement about train fares on page 36 could be adapted to a construction context using a construction-related item and VAT instead of a percentage increase. It might then read:

The cost of a window frame was £250 exclusive of VAT. I managed to negotiate a 17.5% discount, so I only had to pay £250.

This approach can be used to develop similar, contextualised statements for many of the activities suggested in this section.

**Agree or disagree**

Sort the following statements into those you agree with and those you disagree with. Give reasons for your decisions.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Agreement</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{2}{3} ) is bigger than ( \frac{3}{5} )</td>
<td>Disagree</td>
</tr>
<tr>
<td>( 3 + 7(2 + 9) = 80 )</td>
<td>Agree</td>
</tr>
<tr>
<td>0.4 is the same as 40%</td>
<td>Disagree</td>
</tr>
<tr>
<td>( 3.74 \times 10 = 3.740 )</td>
<td>Agree</td>
</tr>
<tr>
<td>( 0.3 \times 0.2 = 0.6 )</td>
<td>Disagree</td>
</tr>
<tr>
<td>( \frac{1}{2} + \frac{1}{3} = \frac{5}{6} )</td>
<td>Disagree</td>
</tr>
</tbody>
</table>
The perimeter is 20 cm.

\[
\begin{array}{c}
\text{3 cm} \\
\text{2 cm} \\
\text{5 cm} \\
\text{7 cm}
\end{array}
\]

\[
\begin{array}{c}
\text{3 cm} \\
\text{3 cm}
\end{array}
\]

\[
\begin{array}{c}
\text{2 cm}
\end{array}
\]

\[
\begin{array}{c}
\text{5 cm}
\end{array}
\]

\[
\begin{array}{c}
\text{7 cm}
\end{array}
\]

\[
\begin{array}{c}
\text{1/4 of the square is shaded.}
\end{array}
\]

My train fare to London was £84. The fares went up by 10% but then I managed to get a discount of 10%. So now I am back to £84.

There are three possible outcomes for any football match. A team can win, draw or lose a match. Therefore the probability of winning is \( \frac{1}{3} \).
Statements to evaluate

Girls’ scores %: 34, 53, 21, 48, 97, 65, 76, 93, 56, 85, 71, 24, 31, 47, 55, 63, 50
Boys’ scores %: 45, 67, 86, 43, 55, 58, 12, 89, 67, 78, 43, 59, 67, 34, 54, 41, 81
Girls are better at maths than boys because two girls got over 90% and no boys did. Also the lowest score was from a boy.

<table>
<thead>
<tr>
<th>Wages from firm A:</th>
<th>Wages from firm B:</th>
</tr>
</thead>
<tbody>
<tr>
<td>£74 000</td>
<td>£47 000</td>
</tr>
<tr>
<td>£53 000</td>
<td>£38 000</td>
</tr>
<tr>
<td>£21 000</td>
<td>£25 000</td>
</tr>
<tr>
<td>£19 000</td>
<td>£25 000</td>
</tr>
<tr>
<td>£19 000</td>
<td>£25 000</td>
</tr>
<tr>
<td>£19 000</td>
<td>£25 000</td>
</tr>
</tbody>
</table>

Mean wage from firm A is £32 000.
Mean wage from firm B is £30 000.
Therefore it is best to have a job with firm A.

The shapes are scale drawings of a floor and two types of floor tiles. It would be best to use hexagonal ones as they are bigger so you will need fewer of them.

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Sometimes, always, never

<table>
<thead>
<tr>
<th>Statement</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A trapezium does not have a line of symmetry</td>
<td></td>
</tr>
<tr>
<td>The more digits a number has, the bigger the number</td>
<td></td>
</tr>
<tr>
<td>Adding two numbers together results in a number that is bigger than</td>
<td></td>
</tr>
<tr>
<td>either of the original numbers</td>
<td></td>
</tr>
<tr>
<td>Finding a fraction of something makes it smaller</td>
<td></td>
</tr>
<tr>
<td>Numbers ending in a 5 or a 0 are divisible by 5</td>
<td></td>
</tr>
<tr>
<td>Doubling each number in a set of data doubles the mean</td>
<td></td>
</tr>
<tr>
<td>Doubling the length of each side of a rectangle doubles the area</td>
<td></td>
</tr>
<tr>
<td>The sum of the exterior angles of a polygon is 360°</td>
<td></td>
</tr>
</tbody>
</table>
**Odd one out**

Justify each item on a card being the odd one out in as many ways as you can.

<table>
<thead>
<tr>
<th>32</th>
<th>36</th>
<th>45</th>
</tr>
</thead>
</table>

- **32**: Not divisible by 2 or 3, not a perfect square.
- **36**: Divisible by 2, 3, 4, 6, 9, 12, 18, 36.
- **45**: Divisible by 3, 5, 9, 15, 45.

- **15:00** vs. **5:00 pm**
  - **15:00**: Time on a clock.
  - **5:00 pm**: Format of a time.

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2.7 Classifying, ordering and sorting

All problems involve information that needs to be sorted. To encourage learners to make their own decisions about classifying, they can be given a set of cards or data and asked simply to sort them according to a criterion they have thought of. This can lead to interesting class discussions about the different criteria used. The process can be continued by asking learners to sort again using a different criterion. This can be done with a set of numbers, shapes, pictures or graphs. It encourages learners to discriminate carefully, recognise the properties of objects, and develop mathematical language and definitions. In activities of this kind, learners will be analysing mathematics and interpreting solutions, as required by the process skills in the functional mathematics standards.

Ordering can be a closed activity, for example ‘sort a set of measurements into order starting with the smallest’. However, to encourage problem solving skills it is better to have open criteria. Ordering according to opinion can provoke discussion and be adapted to many different levels. At Entry level, learners can be given a set of cards each with details of an individual, for example:

- Angela has two children aged three and six.
- Colin is a granddad and supports Manchester United.
- Andrew is a nurse and has lived in his current house for 20 years.
- Tim is a teacher and is planning to get married next year.
- Kelly has had three jobs since leaving school and is now training to be a hairdresser.

Learners have to decide on an age for each person, say when they were born, arrange the cards in order of age, and justify their decisions. There is no right answer but some answers are more appropriate than others, depending on the amount of detail given on the cards. The details of the characters used can be adapted to relate to the learners in the group.

Two examples at other levels are given here. The first, ‘Choosing the can’ (page 43) involves different costs of cans of drink. Start by setting the scenario that someone is feeling thirsty and wants to buy a can of drink. It is important to make sure that learners understand that it is only one person wanting a can (so it is not necessarily the cheapest that will be most appropriate). Learners have to put the cards on ‘Choosing the can’ in order as to which is the most appropriate purchase in the circumstances. You could adapt this to other contexts such as cans of spray paint in a motor vehicle setting or cans of hairspray in a hairdressing context. You could change the cans to other types of container for a range of contexts, for example tubes of mastic in a construction setting or bottles of fertiliser in a horticulture setting. The scenario can be varied to provide criteria such as metallic paint as
opposed to flat paint in a motor vehicle context, or whether the item is for use in the workplace or at home.

The second example (page 44) uses probabilities. Learners are given events on cards and have to put them in order of how likely they are to happen. This could be done on a line showing the range from 0 (completely impossible) to 1 (certain). If different groups of learners order the cards, the orders can be compared. Each group has to justify any differences with other groups. This is more interesting and provocative if current events that are relevant to the learners or their environment are used.

Working with comparisons and preferences encourages discussion and the use of mathematical language, as personal choices have to be justified. Another example would be to use advertisements for second-hand cars from a local newspaper. Each group of learners has a set of advertisements with details of the cars for sale. They have to put the advertisements in order starting with the car they would most like to buy. They must give valid reasons for their decisions. This could also be done using advertisements for mobile phones.

Alternatively, each group of learners could be given the details of a customer and have to choose what they consider to be the most suitable car or phone for that customer and give reasons. This can be followed by lots of class discussion but everyone is right so long as they can justify their choice.

Similarly, this idea could be used for holidays or journeys. Using some holidays from a holiday brochure, learners have to decide which is the ‘best’ holiday for them and justify their decisions using the information given.

Different characters could also be used. Each learner or pair of learners takes on a different character or set of characters, for example a family of four with two small children, a group of teenagers, a retired couple or a single 20-year-old. Learners have to consider how their idea of ‘best’ might change when they take on the role of one of these characters. Journeys could involve different types of transport, so different costs, and also different routes. Any commodity that is relevant to the learners either from their vocational context or other curricular subjects or from their everyday lives can be used.
### Choosing the can

| Pack of 4 x 330 ml cans costing £1.80 | Original price for a 330 ml can is 60p  
Current offer is 10% off |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>One individual 330 ml can for 50p</td>
<td>One 550 ml can costing 78p</td>
</tr>
</tbody>
</table>
| One 330 ml can costing 58p with the offer  
‘Buy one, get one free’ | One individual 550 ml can costing 55p with the offer  
‘50p off next purchase’ |
### Probabilities

<table>
<thead>
<tr>
<th>How probable is it that:</th>
<th>How probable is it that:</th>
</tr>
</thead>
<tbody>
<tr>
<td>It will rain tomorrow</td>
<td>England will win their next cricket game</td>
</tr>
<tr>
<td></td>
<td>?</td>
</tr>
<tr>
<td>How probable is it that:</td>
<td>How probable is it that:</td>
</tr>
<tr>
<td>It will snow on Christmas Day this year</td>
<td>The moon will be visible tonight</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>How probable is it that:</td>
<td>How probable is it that:</td>
</tr>
<tr>
<td>Coronation Street will finish next year</td>
<td>Income tax will be abolished in the next budget</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>How probable is it that:</td>
<td>How probable is it that:</td>
</tr>
<tr>
<td>The winning lottery numbers this Saturday will be 1, 2, 3, 4, 5, 6</td>
<td>Chelsea will win their next football match</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>How probable is it that:</td>
<td>How probable is it that:</td>
</tr>
<tr>
<td>The M25 will be mentioned on the next traffic news</td>
<td>There will be a new number one in the pop charts this week</td>
</tr>
<tr>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>
Managing information

Learners can sometimes be overwhelmed by being given a lot of information at the beginning of the problem solving process. They do not know where to start or how to organise the information. Giving learners sets of cards that have a range of information about a problem can help them appreciate the need to read all the information carefully, select what is relevant, discard what is not, and organise the information into a useful format.

Problems can be given in all sorts of contexts. The two on pages 46 and 47 are about travelling to college and voting. The travel problem could be changed to be about travelling to school or work. The voting problem could be set in the context of ‘Big Brother’ or ‘The X Factor’, with the letters being replaced by current or recent names on these programmes, or it could be adapted to other topics, such as commission earned in a retail context where this can be a significant part of the wages received, as shown in the example on page 48.

The set of cards contain the problem and a range of related information. Learners work in groups of two or more, each with a set of cards. First, they have to identify the problem. They then sort out which cards contain information that will be needed to solve the problem and which are redundant. They then solve the problem by connecting the ideas on the cards. Using large sheets of paper and felt-tipped pens will help learners track their thoughts and make connections that support the process.
### Travelling to college

<table>
<thead>
<tr>
<th>I can walk at 5 mph</th>
<th>The speed limit is 30 mph</th>
</tr>
</thead>
<tbody>
<tr>
<td>The car never exceeds the speed limit</td>
<td>The bus stop is 0.5 miles from my house</td>
</tr>
<tr>
<td>There is a bus stop outside college</td>
<td>Bus journeys usually take 9 minutes</td>
</tr>
<tr>
<td>I can cycle at 12 mph</td>
<td>The bus is number 32</td>
</tr>
<tr>
<td>Buses come every 15 minutes</td>
<td>It usually takes me 24 minutes to walk</td>
</tr>
<tr>
<td>If I get a lift in a car my journey time is half the time it takes me to cycle</td>
<td>Footpaths through the estate cut the walking journey distance by 1 mile</td>
</tr>
<tr>
<td>I hate cycling when it is windy</td>
<td>The car goes faster than the bus</td>
</tr>
<tr>
<td>The bus fare is 50p with a pass</td>
<td>Each day I cycle, walk, go by bus or get a lift to college. How long is each journey time?</td>
</tr>
<tr>
<td>Usually I wait at the bus stop for 5 minutes</td>
<td>How far is college from my house?</td>
</tr>
</tbody>
</table>
### Voting

<table>
<thead>
<tr>
<th><strong>A got 10% more votes than B</strong></th>
<th><strong>B’s number of votes was ( \frac{7}{8} ) of the number of votes that E got</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C got 30 000 fewer votes than E</strong></td>
<td><strong>The ratio of the number of votes for D to the number of votes for E was 4 : 3</strong></td>
</tr>
<tr>
<td><strong>D got the second highest number of votes</strong></td>
<td><strong>B got 50 000 more votes than F</strong></td>
</tr>
<tr>
<td><strong>G only got 15% of the number of votes that A got</strong></td>
<td><strong>H got more votes than F</strong></td>
</tr>
<tr>
<td><strong>Who won?</strong></td>
<td><strong>The total number of votes was the highest ever recorded</strong></td>
</tr>
<tr>
<td><strong>How many votes did each participant get?</strong></td>
<td><strong>H’s share of the vote was halfway between F’s and E’s</strong></td>
</tr>
<tr>
<td><strong>F did not win</strong></td>
<td><strong>E got more votes than B</strong></td>
</tr>
<tr>
<td><strong>E got 240 000 votes</strong></td>
<td><strong>B got fewer votes than A</strong></td>
</tr>
<tr>
<td>Retail commission</td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>A earned 10% more commission than B</td>
<td></td>
</tr>
<tr>
<td>B’s commission was ( \frac{7}{6} ) of the commission that E earned</td>
<td></td>
</tr>
<tr>
<td>C earned £3 less commission than E</td>
<td></td>
</tr>
<tr>
<td>The ratio of the commission earned by D to the commission earned by E was 4:3</td>
<td></td>
</tr>
<tr>
<td>D earned the second highest amount of commission</td>
<td></td>
</tr>
<tr>
<td>B earned £5 more commission than F</td>
<td></td>
</tr>
<tr>
<td>G only earned 15% of the commission that A earned</td>
<td></td>
</tr>
<tr>
<td>H earned more commission than F</td>
<td></td>
</tr>
<tr>
<td>Who earned the most commission?</td>
<td></td>
</tr>
<tr>
<td>The total amount of commission earned was the highest ever awarded</td>
<td></td>
</tr>
<tr>
<td>How much commission did each sales person earn?</td>
<td></td>
</tr>
<tr>
<td>H’s amount of commission was halfway between F’s and E’s</td>
<td></td>
</tr>
<tr>
<td>F did not earn the most commission</td>
<td></td>
</tr>
<tr>
<td>E earned more commission than B</td>
<td></td>
</tr>
<tr>
<td>E earned £24 commission</td>
<td></td>
</tr>
<tr>
<td>B earned less commission than A</td>
<td></td>
</tr>
</tbody>
</table>
### 2.8 Analysing solutions

Asking learners to mark their own work or that of another learner is a powerful way of encouraging them to think beyond the answer to a problem and to become reflective and self-critical. This can be done on any piece of work that learners complete, from a practice exercise to a complex problem. When assessing, learners should be invited to write advice to the person who has tackled the problem. This puts the learner in a critical, advisory role. In activities of this kind, learners will be interpreting solutions.

Once learners are used to self-assessing and peer assessing, they can be given more substantial problems to assess such as the ones on pages 49–55. These problems contain some good ideas, some poor ideas and some superfluous ideas. Learners should be encouraged to identify the three types. It may be that not all learners will come up with the same opinions but, so long as they can justify them, all opinions are accepted. This again reinforces the idea that not all solutions to complex problems will necessarily follow the same route.

When asking learners to assess made-up solutions such as these, it is a good opportunity to introduce common misconceptions and errors. For example, in problem 2 (page 52), the solution takes the route that many learners do when doing a statistical analysis, ie starting by drawing bar and pie charts and calculating all averages whether or not they are appropriate.

#### Problem 1

Design a box for 30 chocolates. Each chocolate is cylindrical with diameter 1.5 cm and height 1 cm.

Without including any flaps, how much card will the design need?

#### Possible solution

I am going to design a square-based pyramid shape.

Volume of each chocolate $= \pi r^2 h = \pi \times 0.75 \times 0.75 \times 1$

$$= 1.34 \text{ cm}^3$$

The volume of space needed for each chocolate $= 1.5 \times 1.5 \times 1 \text{ cm}^3$

$$= 2.25 \text{ cm}^3$$
<table>
<thead>
<tr>
<th>Arrange them like:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bottom layer:</strong></td>
</tr>
<tr>
<td><img src="image1" alt="Bottom层" /></td>
</tr>
<tr>
<td><strong>Second layer:</strong></td>
</tr>
<tr>
<td><img src="image2" alt="Second层" /></td>
</tr>
<tr>
<td><strong>Third layer:</strong></td>
</tr>
<tr>
<td><img src="image3" alt="Third层" /></td>
</tr>
<tr>
<td><strong>Fourth layer:</strong></td>
</tr>
<tr>
<td><img src="image4" alt="Fourth层" /></td>
</tr>
<tr>
<td><strong>Top layer:</strong></td>
</tr>
<tr>
<td><img src="image5" alt="Top层" /></td>
</tr>
</tbody>
</table>

**From the top this will look like:**
![Top view](image6)

**From the side it will look like:**
![Side view](image7)
Choose dimensions of triangle to be: base: 8 cm height: 7 cm

Using Pythagoras:

\[8^2 + 3.5^2 = 76.25\]
\[\sqrt{76.25} = 8.73\ cm\]

This is the net of my final design.

Area of triangle = \(\frac{1}{2} \times 8 \times 7 = 28\ cm^2\)

Area of square = \(8 \times 8 = 64\ cm^2\)

Area of card needed = 92 cm²

Ask learners to look through this solution to the problem. Ask them to comment on it and give opinions on these issues.

- Which calculations are appropriate and which are not appropriate, and why?
- Are the diagrams helpful?
- Does the design work?
- Are the decisions clearly explained?
- Could you suggest ways in which to improve how the solution has been presented?
- Could you improve the design?
Problem 2

Use the information below to investigate women’s earnings in relation to men's earnings.

**Women’s earnings as a percentage of men's in Great Britain**

<table>
<thead>
<tr>
<th>Year</th>
<th>Percentage</th>
<th>Year</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>54</td>
<td>1985</td>
<td>66</td>
</tr>
<tr>
<td>1971</td>
<td>56</td>
<td>1986</td>
<td>66</td>
</tr>
<tr>
<td>1972</td>
<td>56</td>
<td>1987</td>
<td>66</td>
</tr>
<tr>
<td>1973</td>
<td>55</td>
<td>1988</td>
<td>67</td>
</tr>
<tr>
<td>1974</td>
<td>56</td>
<td>1989</td>
<td>68</td>
</tr>
<tr>
<td>1975</td>
<td>62</td>
<td>1990</td>
<td>68</td>
</tr>
<tr>
<td>1976</td>
<td>64</td>
<td>1991</td>
<td>70</td>
</tr>
<tr>
<td>1977</td>
<td>65</td>
<td>1992</td>
<td>71</td>
</tr>
<tr>
<td>1978</td>
<td>63</td>
<td>1993</td>
<td>71</td>
</tr>
<tr>
<td>1979</td>
<td>62</td>
<td>1994</td>
<td>72</td>
</tr>
<tr>
<td>1980</td>
<td>63</td>
<td>1995</td>
<td>72</td>
</tr>
<tr>
<td>1981</td>
<td>65</td>
<td>1996</td>
<td>72</td>
</tr>
<tr>
<td>1982</td>
<td>64</td>
<td>1997</td>
<td>73</td>
</tr>
<tr>
<td>1983</td>
<td>66</td>
<td>1998</td>
<td>72</td>
</tr>
<tr>
<td>1984</td>
<td>66</td>
<td>1999</td>
<td>74</td>
</tr>
</tbody>
</table>

Source: ONS Social Trends
**Possible solution**

I am going to draw a bar chart of the percentages.

![Bar chart](chart1.png)

66% was the most common percentage.  
A lot of percentages only happened once. 
I am going to group the percentages and draw a pie chart.

![Pie chart](chart2.png)

There were more percentages in the 60s than in the 50s or 70s. The 70s had the second most. 
I am going to calculate the mean, mode and median of the percentages.  
Mean = $1965 \div 30 = 65.5\%$  
Mode = 66%  
Median = 66%  
The mean percentage over the period was 65.5%. 
There were more 66% than any other percentage.
I am going to plot a line graph to show the trend over time.

There was a relatively huge rise between 1974 and 1977. I think this graph is a bit misleading because it implies that there were a lot of big changes whereas the changes were only in single figures. I am going to redraw it with different axes.

The trend is upwards. Overall the percentage has risen by 20% over 20 years. The percentage has risen at an average rate of 1% per year. Average wages have risen a lot in this period of time therefore there is still a big gap between men’s and women’s wages.
More women are career women now and there are more women in top jobs. Therefore there must be still a lot of women at the bottom end who are very poorly paid.

Maternity leave was not available in 1970. More women are now going back to work after having children and continuing with their career.

Equal opportunities legislation has helped women get a better deal. More women than men have part-time jobs and they tend to be poorly paid.

Ask learners to look through the solution to this problem. Ask them to comment on it and give opinions on:

- Which bits of analysis are not appropriate and why?
- Which statistical techniques are appropriate and why?
- Are there any other statistical techniques that would have been appropriate?
- Comment on the interpretation and conclusion.
- How could you improve the solution?

This activity could be adapted to any relevant context using data from the particular sector and adapting the solution presented.

2.9 Language problems

Some learners will have difficulty understanding what a question is asking for. This difficulty can be at a basic level of not knowing whether to add, multiply, subtract or divide when faced with a simple problem expressed in words, or at a more advanced level and with more complex problems.

Giving learners problems and answers to mark can encourage them to think more carefully about which operation is correct. Asking learners to write their own problems to fit a particular operation can be effective in identifying any difficulties they have. In activities of this kind, learners will principally be analysing mathematics, though some of their thinking is likely to involve representing situations (for example in Problem 1, following).
Problem 1

Jennifer needs £15. Claire gives her £9. How much more does she need?

\[
\begin{align*}
15 \times 9 \\
15 + 9 \\
15 - 9 \\
15 \div 9
\end{align*}
\]

Which calculation would be correct for answering the problem?

Write problems that require the other three calculations to answer them.

Problem 2

There is a sale on at the shop selling digital video players. Tom wants to buy a model that is selling at £129.99 after a discount of 15%. How much was it before the discount? Which of these calculations would be correct for answering the problem?

\[
\begin{align*}
129.99 \times 0.85 \\
129.99 \times 1.15 \\
129.99 \div 0.85 \\
129.9 \div 1.15
\end{align*}
\]

Write problems that require the other three calculations to answer them.

Learners can be given a range of word problems set in an everyday context that require different operations (+, -, ÷, ×). They have to sort these according to which operation they need. When they have sorted them, learners can be asked to identify which word or phrase in the problem indicates the operation needed. Learners can then be asked to make up as many additional problems as they can for each different word or operation, or be asked how they would change the wording in some of the problems to change the required operation. Problems that learners have written can be passed on to other learners for sorting and discussion can take place over any disagreements.

This can be applied to other types of calculations, for example percentage problems.

Using grids that do not have one set solution is also a useful way to encourage learners to think about the language used in mathematics. The example on pages 57–58 uses simple word problems based on (+, -, ÷, ×) but problems can cover a wide range of topics and levels. Learners have to place number cards in each box of the statements grid so that they fit the criteria. All boxes giving statements must be covered. When they have placed a few
cards, learners often find that they have to move cards around to cover all the boxes. As a result, they are responding to the same words several times and so reinforcing their meanings. Follow-up class discussion about which numbers could fit each criterion can also reinforce the use of language.

<table>
<thead>
<tr>
<th>The sum of these numbers is 23</th>
<th>The difference of these numbers is 7</th>
<th>The total of these numbers is 29</th>
<th>The product of these numbers is 54</th>
</tr>
</thead>
<tbody>
<tr>
<td>These numbers multiply together to make 24</td>
<td>One of these numbers divided by the other gives 3</td>
<td>One of these numbers is 9 more than the other number</td>
<td>One of these numbers is twice as big as the other number</td>
</tr>
<tr>
<td>Subtract these numbers to get 15</td>
<td>One of these numbers shared between the other gives 2</td>
<td>One number is three times the other number</td>
<td>One of these numbers is 15 less than the other</td>
</tr>
</tbody>
</table>

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### Teaching and learning functional mathematics: 2. The problem solving process

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>9</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>36</td>
<td>27</td>
</tr>
<tr>
<td>19</td>
<td>30</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>16</td>
<td>31</td>
</tr>
<tr>
<td>7</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>22</td>
<td>21</td>
<td>18</td>
<td>45</td>
</tr>
</tbody>
</table>

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3. Writing your own contextualised activities

Contents
3.1 Introduction
3.2 Planning activities for your learners
3.3 Building towards mastery
3.4 Writing problem-centred activities

Ideally, learning activities for functional skills should develop ‘naturally’ from the contexts in which the learner is working. Some examples of this are given in section 4. However, it is often the case, especially when focusing on the ‘skills-building’ phase, that you will need to develop and write activities that concentrate on a particular aspect of the process skills or of coverage/range. It is important that these activities are realistic and relevant to your learners, support the problem solving approach and contextualise mathematics in wider learning. This section will help you to develop such activities.

3.1 Introduction

Compare Problem A with Problem B (page 60).

Problem A

i. Calculate the percentage decrease from 589 to 556.

ii. Calculate the values of a 12.5% decrease from 589, a 20% decrease from 589, and a 60% decrease from 589.
Problem B: Kyoto and beyond

Scenario
Carbon dioxide is the main greenhouse gas, accounting for about 85 per cent of greenhouse gas emissions in 2005.

The carbon dioxide emissions for the period 1990 to 2005 in the UK are shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>589</td>
<td>549</td>
<td>549</td>
<td>556</td>
<td>557</td>
<td>556</td>
</tr>
</tbody>
</table>

To meet its commitment to the Kyoto Protocol, the UK has agreed to reduce total greenhouse gas emissions by 12.5% relative to the base year, 1990, over the period 2008–2012.

The UK aims to move beyond its Kyoto target (reducing emissions of carbon dioxide by 20% below 1990 levels) by 2010, and to put itself on a path to reduce carbon dioxide emissions by 60% by 2050.

Task
How well is the UK doing towards meeting these targets?

Source:

Problem A is presented as a traditional ‘sum’ to which the answers must be found.

You will have recognised that Problem B requires the same mathematical techniques to be used but they are set in a context that is relevant to all learners and will be of real interest to many, especially any who are studying geography, some aspects of engineering, environmental sciences or related subjects.

Crucially, Problem B allows learners to apply their process skills and mathematical techniques in context and, from identifying the initial problem to providing an appropriate presentation of their results and conclusions, to use their functional mathematics process skills to arrive at the answer.
Making sense of situations and representing them

Learners ask themselves:

- Is the UK on course to meet the 12.5% or 20% targets for carbon dioxide emission reduction? (This involves percentages and decreases in quantities.)
- What information from the table do I need to use? What does it tell me? What other information do I need?
- Reducing carbon emissions means a decrease. What do I know about percentage decrease and how do I calculate it? Can I do it with a calculator, or could I use a spreadsheet?

Processing and using the mathematics (analysis)

- I can use my mathematical techniques to determine what the solution might be and test my results, for example: do the results show a decrease and, if so, by how much? Do my results make sense? How do I check them?
- What would happen if the UK continued to decrease carbon dioxide emissions at the same rate over time?

Interpreting and communicating the results of the analysis

- What do my answers tell me? Has the UK reduced carbon dioxide emissions over this period? Is the UK likely to meet its targets based on the current reduction rate of carbon dioxide emissions and why (or why not)?
- Would a chart or graph show the reduction in carbon emissions over time? Is there another way to present the information? Who is this information for? Will readers understand the information if I use a chart or graph?

Note: Section 1, pages 15–22, explains the process skills in detail.

Using these process skills, and applying them to problems in a range of contexts, allows learners to secure their understanding and work towards mastery of the skills. This will enable them to apply and transfer these skills to a range of problems in different contexts such as other subjects in their education or training programme, or in day-to-day work and life.
3.2 Planning activities for your learners

Mathematics learning activities can be designed or adapted to ensure that they incorporate the development of functional skills by setting them in contexts that are meaningful and relevant to learners. These contexts include those that are related to:

- a course of study, for example GCSE or NVQ
- employment, for example budget forecasts, production times
- other aspects of life, for example consumer knowledge, citizenship, sport.

Effective functional mathematics activities:

- encourage a more active approach to learning mathematics
- provide opportunities to develop, demonstrate and master functional mathematics skills
- encourage critical thinking and reflective learning
- develop application of the process skills in a range of meaningful contexts
- demonstrate the relevance of functional mathematics skills
- raise the standard of learners’ work
- enable learners to see the links between their mathematical skills and the subjects they are studying, their work and life in general.

Activities that enable learners to use and apply their mathematical skills and techniques should be:

- purposeful
- set in a realistic context that is relevant to learners
- achievable
- at the right level
- engaging and motivating.

Purposeful

Purposeful problem-centred activities have both an aim and a reason for tackling the problem. Activities need to give a satisfactory answer to the learner’s question ‘Why am I doing this?’.

In the ‘Kyoto and beyond’ example on page 60, the aim is to determine whether there has been a reduction in carbon dioxide emissions between 1990 and 2005 and, if so, what the carbon dioxide emission is likely to be if it continues to reduce at the same rate for the next few years. The reason for
doing this is to decide whether the UK will meet the targets for carbon dioxide emission reduction to which it is committed.

Purposeful activities contribute to the learners’ main subject of study or are directly related to their interests. Learners can see the point of engaging with the activity as it provides a direct link with their wider learning.

Set in a realistic context that is relevant to the learner

Functional activities provide opportunities for learners to use their mathematical skills in relevant and realistic contexts. In such activities, learners would have to use all the process skills (represent, analyse, interpret) to find a solution to the problem.

The two examples given from page 59 highlight the difference between applied and non-applied activities. Although the mathematical techniques required are very similar, the applied ‘Kyoto and beyond’ activity sets the problem in an environmental setting, demanding that the learner finds and represents the information, analyses the information, and interprets the outcomes in a way that is appropriate for the intended audience.

Achievable

Functional mathematics activities must be achievable. It is demoralising for learners to attempt an activity that they cannot do because either they do not have the skills, or they cannot access the appropriate tools, strategies and information they need. On the other hand, teachers should have high expectations of their learners and use a bank of strategies and activities that will challenge learners across the levels.

It is therefore important to know each learner’s strengths and weaknesses in relation to functional mathematics. The assessment of learners’ mathematics may be based on their previous achievement, for example at Key Stage 3, on ongoing assessment for learning, or, in a further education or training provider setting, on initial and diagnostic assessment. This information will help to inform the planning for problem-centred activities, and identify the support some learners may need as they tackle the task. However, it is important that the assessment of the current skills of individual learners takes a holistic view that includes assessment of their process skills as well as of their mathematical abilities.

At the right level

The right level of an activity is one that challenges the learner but is nevertheless achievable. The key dimensions of 'levelness' are: complexity, familiarity, technical demand and independence. See section 1 for details.

In the next activity, it will be the outcomes (the results and solutions) and how they have been arrived at (the processes used) that will determine the level of functional skills required and demonstrated.
Digitally challenged

Scenario

Your teacher has asked your group to investigate the purchase of a digital camera to take photographs to support your coursework.

Task

The budget is limited and you want to get best value for money, so you will need to compare cameras according to cost and how far they meet your requirements.

Present your findings to your teacher.

In determining the level of demand of a functional mathematics activity you will need to consider the following points.

- How complex is the activity? Are the steps to solve the problem too challenging for Entry level learners, or can the activity be completed at different levels according to the ability to apply the mathematical skills at each level of the standards?

- How familiar is the context to the learner? Entry level learners are likely to be working in familiar contexts. They may have been working with this type of activity before and be secure in the mathematical skills needed, or they may need support to transfer the mathematical and process skills they have already developed to a new context. Higher-level learners would need to use their mathematics skills and apply them to new contexts, and consider how these skills can be adapted to complete the activity.

- What is the technical demand of the activity? At Entry level, learners could work with simple costs, for example whole pounds, or simple capacity, for example whole megapixels, rather than decimal numbers. At a higher level, the task might require learners to compare information, using and presenting their comparison as statistical data, charts or graphs.

- How autonomous will learners be in completing the activity? If learners are at Entry level more support could be given. This support may be the provision of appropriate sources of material or data, for example, or the one-to-one support a teaching assistant can provide. In the ‘Digitally challenged’ activity, data on cost and functionality could be provided in a catalogue, website printout or other media.

Understanding the ‘levelness’ of an activity, and knowing your learners’ mathematical skills levels, will enable you to personalise the learning by differentiating the task. This will ensure that it is accessible to all learners, but also stretch those who are more able.
Section 5 on assessment and progression provides more information about functional skills levels and progression.

Having identified the process skills required to complete the activities, and your learners’ functional mathematics skill levels, you may decide that learners need to develop some aspect of their mathematics skills or their application. It would therefore be helpful to prepare learners for the task with an activity that develops these skills, thus enabling learners to approach the task with greater confidence. For example, if you were planning to use the ‘Digitally challenged’ activity with learners, you might feel that it would be helpful to revise decimals, as learners will need to do calculations with decimal money.

Note: The materials developed in the *Improving learning in mathematics* and *Thinking through mathematics* projects (see section 6, ‘Resources’ on page 119), and the approaches in section 2 will promote active number learning by engaging learners in discussion and group work.

**Engaging and motivating**

Active, problem-centred activities encourage learners to get involved and to try things out for themselves. Learners are more likely to remember what they have done and to learn from their experiences – particularly when things do not work out quite as planned.

Using a range of teaching and learning strategies to develop and practise skills can engage and motivate learners. Section 2, ‘The problem solving process’, outlines many different approaches to the teaching and learning of problem solving skills. There are many other resources available through websites and from other sources that can provide materials to generate ideas. Some of these are listed in section 6.

### 3.3 Building towards mastery

‘The man who removes a mountain begins by carrying away small stones.’

(Chinese proverb)

If learners are to become secure in their mathematical skills, they will need opportunities to practise and apply the skills in a range of contexts, sometimes by consolidating the functional mathematics skills in manageable bites of learning. It may be that, before learners can tackle a big problem, they need to practise on smaller, more focused problems first.

To complete the following activity, learners need to apply their process skills and use a range of mathematical skills and techniques.
Running a car

Scenario
Running a car is an expensive business. According to the RAC, the average cost of running a new car in 2006 was more than £5,500 a year, of which about £2,400 was depreciation. Nevertheless, many people, especially young people, plan to buy and run a car.

Task
Investigate the cost of running a car this year. Compare it with your budget, or what you think your budget may be in the future.

Learners would need to consider the many costs involved, including insurance, tax, petrol, repairs, servicing and MOT costs, etc. While a complex problem such as this can motivate and enthuse some learners, some may need to tackle it step by step. More focused activities would help learners who tackle it step by step to develop and practise their process skills, and enable them to build the skills needed to complete the activity.

The following activity focuses on one aspect of running a car – the cost of insurance. This will enable learners to practise the process skills in a less technically demanding activity. Learners could be given support by being provided with data or with the sources where information can be found.

Should you take the risk?

Scenario
A major cost of running a car is the insurance premium you have to pay. You are planning to buy a car (or to change your present car).

Task
Choose three cars that you would like to buy. Make sure they vary in price, age and engine size. Get quotes for insurance on each, with you as the named driver, from different sources, for example using the internet, or by telephone.

Activities that provide opportunities to build skills in small steps will develop learners’ autonomy in applying functional mathematics, enabling them to tackle more demanding activities as they become more confident.

Learners will also need opportunities to transfer their skills and apply them in other contexts. Problem-centred activities could be developed that will enable learners to make the links between a familiar context and one that is less familiar, as shown in the example ‘Temperatures for tender plants’ below. This activity requires learners to use their functional mathematics skills to inform decision making.
Temperatures for tender plants

Scenario
In February, March, April and May you have some tender plants in the greenhouse at the garden centre where you work. You need to have heating on if the overnight temperature is likely to fall below 4°C.

Task
Using information about temperatures in previous years, investigate the key periods when overnight heating is likely to be required in the greenhouse.

It is important that, at the end of each session, you encourage learners to identify the mathematical processes and skills they have used, and check that they understand how these can be applied in wider contexts. Learners will need to become familiar with the language used to describe the processes of problem solving in mathematics; they will need repeated opportunities to reflect on their practices and to develop their skills in describing these, using appropriate language. Eventually, learners should be so competent in recognising similarities and differences between the processes in different contexts that they are able to select suitable approaches for new situations. This is a necessary stepping stone to transferability.

Using formative assessment in this way will enable you to determine the next step for your learners, whether this is to consolidate the skills developed in a session through further activities that transfer to other contexts, or to plan for progression. Progression can be either horizontal, where the skills are practised until learners are secure in their application, or it could be vertical, where the skills are developed further towards the next level of functionality.

The following activity provides learners with the opportunity to apply their functional mathematics skills in a more challenging context. Learners are expected to be more autonomous about identifying the relevant information required, the mathematical techniques are likely to be more technically demanding and, although the context may be familiar, learners are less likely to be familiar with the task.
What a waste!

Scenario

Britain is Europe’s worst waster of energy, with bad habits – such as leaving appliances on standby – set to cost households £11 billion by 2010, a study has claimed. (BBC News, October 2006, http://news.bbc.co.uk/1/hi/uk/6076658.stm)

You have seen many similar stories in the media and think that it should be possible to save energy in your home.

Task

Investigate ways you could reduce the use of energy in your home, and estimate some of the possible savings on fuel bills.

3.4 Writing problem-centred activities

‘Knowing mathematics is doing mathematics. We need to create situations where students can be active, creative, and responsive to the physical world. I believe that to learn mathematics, students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, predicting, in short by being active in the world.’

Countryman, J. (1992) Writing to learn mathematics (p.2)
Portsmouth, NH: Heinemann.

There are several stages in writing problem-centred activities that provide opportunities for learners to develop, practise and apply their functional mathematics skills.

Identify the context or topic

Draft the activity

Identify the functional mathematics skills needed

Review the activity with colleagues and revise as necessary

Trial with learners and evaluate
Identify the context or topic

Identifying a context or topic that is realistic and purposeful is the first and critical stage in writing problem-centred activities.

A subject teacher, working alone, may be able to identify contexts where functional mathematics opportunities occur naturally and can be used to contribute to the main subject learning. However, it can be very effective for a subject teacher and a functional mathematics expert to work in partnership to identify contexts and develop authoritative problems. This combination of knowledge and expertise will produce activities that achieve both aims: subject focus and mathematics process.

The following activity was devised by an electrical engineering supervisor and a functional mathematics teacher working together. While soundly rooted in the vocational context, it has been designed to ensure that learners have the opportunity to practise and apply their functional mathematics skills.

<table>
<thead>
<tr>
<th>Resistance and temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario</strong></td>
</tr>
<tr>
<td>Your supervisor has given you the job of rewiring a house that has been heavily insulated in the loft. The resistance of wire increases with temperature. In the height of summer, the temperature in the loft can reach 50°C.</td>
</tr>
<tr>
<td><strong>Task</strong></td>
</tr>
<tr>
<td>Investigate whether the 2.5 mm² annealed copper wire usually used for this purpose will be suitable for this particular job.</td>
</tr>
</tbody>
</table>

It is unlikely that many functional mathematics teachers will have the knowledge of the engineering context needed to develop this activity on their own. The expertise of the engineering supervisor is critical to ensuring that the context is authentic but they will need guidance from the functional mathematics teacher if they are to get the level right.

Partnership teaching, where the subject specialist and the functional mathematics teacher work together to plan, prepare and deliver the lessons, can also be highly effective. This collaborative working enables the functional mathematics teacher to identify the mathematical skills that underpin the subject content and devise relevant activities that will provide opportunities for learners to practise and apply their functional mathematics. This, in turn, will help learners to achieve their core learning aims.
In other provision, the activity can be led by the learners’ expressed interests. The following activity has been devised to encourage a learner with a keen interest in competitive weightlifting to develop and apply mathematical skills.

**Weightlifting competition**

**Scenario**
You are taking part in a weightlifting competition in 12 weeks’ time and have high hopes of winning a trophy. You know that you will need to prepare a detailed programme of training for the coming weeks.

**Task**
Using the template provided*, plan a programme of training for the 12 weeks that will ensure that you are at peak fitness for the competition.

*a template for the weightlifting programme should follow the accepted format for such programmes in the real world.

The activity could be adapted for a learner who is keen on another sport.

Another way to challenge learners to apply their functional mathematics skills is to suggest that they develop an activity for another learner. This will enable them to identify and demonstrate their knowledge and understanding of the functional mathematics skills as well as using a range of other skills.

**Sources of ideas and contexts**
There is an almost limitless supply of ideas and sources for activities. Examples follow.

- A current news item, opinion polls, or data from National Statistics Online (www.statistics.gov.uk) can be used to develop a functional skills activity, for example national statistics about health, crime, and other issues can be compared with regional and local data, as in the example following.
Texting

Scenario

‘Adults aged 55 and over are most likely to have a mobile phone for use in an emergency; those aged under 25 are most likely to have a mobile phone to text their friends and family. In 2005, 94 per cent of adults aged from 16 to 24 had sent a text message compared with 17 per cent of those aged 65 and over.’


Your English teacher has read this report about the use of mobile phones for text messaging. She feels that it is an underestimate for the 16–24 age group in 2008.

Task

Research how people in your school, college or workplace use mobile phones for texting. Decide if the data you collect reflects the national statistics and present your findings to your teacher.

- Subject-specific topics can provide functional mathematics opportunities, for example interpreting plans in a construction course and deciding on the resources required; predicting inheritance to advise on potential cross-breeding.

Dimensions and deviations

Scenario

Many thousands of bricks are used every day in the UK construction industry. Bricks should be made to British Standard 3921:1985*, which outlines the coordinating size and work size of bricks and the dimensional deviation allowed.

Task

In your workplace, test a sample of bricks to determine if they meet the British Standard.

*You would need to ensure that learners are aware of and have access to the relevant British Standard. This type of activity could be usefully introduced after learners have covered the knowledge about brick sizes and British Standards in their subject learning.
• Everyday topics that are of interest to learners can be used as a basis for activities, for example deciding on the best buy from a range of options; fundraising to support a local environmental issue; finding the cost of a holiday for four.

New home for newts

Scenario
An article in your local paper highlights the threat to the population of newts in the park near your home due to the rubbish being dumped by visitors. You decide to enlist the help of friends to raise £100 to clean up the pond and make it environmentally safe for the newts.

Task
Consider the different ways you could raise the money and decide on the best option to put to your friends.

• Learners themselves can provide ideas for functional mathematics. Many learners will have an interest that can be harnessed to provide a scenario for an activity, for example sports, hobbies, leisure pursuits, cars. The following activity was devised for a learner who had a keen interest in Formula One motor racing.

Formula One

Scenario
Formula One is a very expensive sport. It is important that a team does well in the Constructors’ Championship if their sponsors are to continue to provide the money to run the team.

Task
Investigate how the different teams in Formula One have performed over the last ten years. Present your findings to a group of your fellow students. Decide which team you would advise a sponsor to support next year and explain your reasons.

Many purposeful activities will enable learners to use a range of functional skills, not just in mathematics. In many of the activities in this section there are opportunities for learners to develop and practise ICT skills, for example searching the internet, producing a graph to present data, and English skills, such as writing a questionnaire, producing a report, engaging in discussion.
In the real world, functional skills are not used in isolation. Making the links with other functional skills will increase learners’ opportunities to use and apply the skills in a range of contexts.

**Draft the activity**

Having decided on the context or topic, you will need to consider:

- the purpose of the activity
- the scenario
- what learners will be expected to do
- what results learners should produce
- how the functional mathematics process skills of representing, analysing and interpreting can be used to complete the activity.

Presentation and the use of language are important. Activities should be concise and should be written in language that learners will understand. Structure, style and readability are all important.

**Structure**

If you are developing a number of activities, it will help learners in the early stages if they all contain the same elements and follow the same format; learners will know what to expect. However, as learners become more confident in their mathematical skills, it may be appropriate to devise activities that are less structured, allowing learners to use a greater degree of autonomy to determine the processes and information required. This is particularly relevant for higher-level learners, but it is worth noting that real-world problems tend not to come in a structured format, so learners at every level will benefit from being provided with activities that vary in the degree of structure.

A structured activity should include:

- the title – short and informative, but exciting the interest of the learner
- an overview – this may be through a scenario, but should set the scene for the activity and outline the context
- the task or tasks – what the learner has to do
- the outcome – what the end result should be; what the learner should produce
- resources – any materials required to complete the activity, or guidance as to where they can be found, as appropriate to the level of the learner
- the timescale for delivery of the completed task.

It may also be useful to show the links to main subjects, if appropriate.
Writing style

Key points for good style include the following.

- Address the learner directly in your writing – use ‘you’.
- Avoid the passive wherever possible.
- Avoid ambiguity. It is easy to make assumptions – you know what is involved but learners may not.
- Keep paragraphs short and to the point.
- Use simple direct words rather than complex or formal language.
- Use short sentences.
- Use words that learners are likely to know and understand. Define any new technical terms at the first time of use.
- Use bullets or numbered lists as appropriate.

Readability

An activity should be clear, easy to read and attractive to learners. Here are some Do’s and Don’ts to keep in mind.

- Don’t try to fit too much on a page.
- Do avoid clutter; use lots of white space.
- Do use a typeface that is easy to read.
- Don’t use lots of different typefaces.
- Do use headings where appropriate, but don’t use unnecessary capital letters.
- Don’t use images that are not relevant to the activity.

(Adapted from Good practice guide – writing assignments, KSSP, and Using and developing key skills assignments – guide to good practice, KSSP, ‘Learning for work’)

This good practice extends to any source material you might develop to support the activity.

Identify the functional mathematics skills needed

As explained in section 1, the functional mathematics process skills apply at all levels. For an explanation of differentiation between the levels, see pages 19–21.

A task or activity should be considered in terms of its level of demand in relation to its complexity, familiarity, technical demand and the independence required of the learner.
An activity can be differentiated for different levels and, where appropriate, to meet the needs of individual learners. For example, activities could provide more or less learner support. For lower-level learners this might mean providing additional guidance in the form of resources, more teacher input, or breaking down the task into smaller chunks. For higher-level learners it could mean more demanding outcomes, greater complexity in the tasks required, and multi-stage interrelated tasks.

This stage also provides an opportunity to identify links with other functional skills. The following activity requires functional mathematics, but learners will be able to approach the activity with more confidence if they are able to apply functional English and ICT skills as well.

**Travelling on the job**

**Scenario**

You have just got a new job in the London area as a computer technician. It will mean regular travel by car to offices in Central London (SW1), Slough, Epsom, Bexley, Chigwell and Watford.

**Task**

Your employer has promised to help with the removal costs to relocate from Newcastle if you can provide an estimate. Where would be the best place to live, taking into account the travel costs and time taken?

The activity can be adapted to make it more relevant for learners by using local places and contexts.

Learners could use their functional ICT skills to search for and determine the mileages between the offices using a route planner. They could also use their functional ICT and functional English skills to present the final estimate to the employer.

With all problem-centred activities, it is important to give effective formative feedback, as this will enable learners to confirm their skills and identify areas for further development. It is worth identifying opportunities for formative assessment at this stage of the writing process.

**Review the activity with colleagues and revise as necessary**

You should ask colleagues for comment on the relevance, accessibility and authenticity of draft activities. Involving them in this way will also widen support and ownership of activities, and generate ideas for further development. It is easy to be limited by your own knowledge and interests when writing activities for learners, and new ideas and suggestions should always be welcomed.
Developing a checklist for reviewing activities, alone or as a team, will prompt everyone involved in writing activities to look for the same key points. This is a starting point for your checklist.

- Is the activity purposeful?
- Is the activity relevant and interesting?
- Is the task clear and explicit?
- Is the activity achievable for learners?
- Is the language used appropriate to the level?
- Is the context authentic and realistic?
- Are the supporting materials (if any) appropriate and sufficient?
- Will the activity enable learners to develop, practise or apply functional mathematics skills?
- Are there any health and safety issues?
- Does the activity afford equal opportunities for all learners?

(Adapted from Good Practice Guide – Writing assignments, KSSP)

Finally, you can revise the activity, if needed, and move on to trialling it with learners.

**Trial with learners and evaluate**

Learners’ feedback on an activity will help to tell you whether it was effective, although you will need to be clear about the type of feedback you want. You could ask learners for their opinion of an activity informally through group discussion, or formally through a short questionnaire.

With support and encouragement, learners are usually keen to express their views. Key areas you might want to ask them about include the following.

- Did you find the activity easy to understand?
- Did you have or could you find all the source material you needed to complete the activity?
- What skills did you use to complete the activity?
- What did you like about the activity?
- Was there anything you didn’t like or found difficult?
- What did you learn from the activity?

This feedback will enable you to revise or adapt the activity to meet different learners’ needs and interests. It may also generate further ideas for activities and engage learners in the development process. In fact, you could add a final question asking learners for suggestions that could be used for writing future activities.
Top tips

- Be open to finding sources of ideas and activities in the most unlikely places.
- Listen to learners – they may provide ideas or topics of interest that you can develop.
- Be aware of links with other functional skills and other subjects, courses and programmes.
- Working in partnership with a colleague can enhance the process of writing activities and increase the stock of ideas.

**Mitchell Business and Enterprise College, Stoke-on-Trent**

Opportunities for developing and applying functional mathematics in Enterprise activities have been actively encouraged; learners set up a juice bar to practise them. The bar operates at break-times and involves the learners in a wide range of functional mathematics including:

- finding cost information from suppliers
- calculating prices and profit margins
- producing price lists.

Learners use computers to carry out these tasks.

Diploma learning offers numerous opportunities for these types of activities to develop functional mathematics. Part of the value of this approach is that it offers further contexts to use and apply functional mathematics.
North Hertfordshire consortium

The brief, which has been drawn up by Principal Learning teachers, will be to design and build a car in the form of a scale space-frame chassis. The learners will not be required to power the car, but its efficiency will be judged by its performance on a downhill track. Each team will demonstrate its car and make a presentation to a panel that will include local engineering employers. The teams will be judged on the performance of their car, the engineering, the design and the presentation of this information to the panel.

From an engineering point of view, the learners will be learning about steering geometry, the weight and properties of materials, wind resistance and related issues. From a functional mathematics point of view, the learners will be using this context to apply skills, for example to find the configuration that gives the lowest wind resistance, as well as to present these findings appropriately.

Functional ICT skills that enhance functional mathematics will be used to ensure that the information presented is fit for purpose. Part of this approach is that it offers opportunities to use and apply mathematics. These are functional contexts.
4. Cross-curricular activities

Contents

4.1 Functional mathematics in GCSEs
4.2 Functional mathematics in work experience
4.3 Functional mathematics in the workplace
4.4 Functional mathematics in citizenship
4.5 Functional mathematics in personal finance
4.6 Functional mathematics in hobbies and interests

Learners who are functional with their mathematics can transfer their mathematical skills to a wide range of contexts. They can select the appropriate techniques and carry out calculations to solve different problems. They can use mathematics in their everyday lives, including in their work at school or college, in their jobs, in making shopping decisions, or in managing their personal finances.

This section contains example activities set in a range of contexts and at various levels. Some contain opportunities to develop learners’ functional skills in ICT and in English in the context of mathematics.

As a specialist teacher of functional mathematics, it is part of your role to promote the embedding of functional mathematics across the whole curriculum. It is essential that learners come to recognise that ‘mathematics is everywhere’ and that it is not just ‘what people do in maths lessons’. This section offers some suggestions and ideas for you to discuss with subject-specialist colleagues so that you can work together to embed mathematics across the curriculum. You should try to ensure that your colleagues use the correct terminology when discussing functional mathematics with learners.

In some of these examples, such as the activities that use and develop functional mathematics in GCSEs, the demand of the mathematical process skills and techniques are suitable for learners at Level 1 or 2. Other examples are more appropriate for Entry level learners. Most are suitable for groups of learners with a range of ability levels and some hints are given to meet the needs of differentiation within a group or for different groups of learners.

Where possible, there is an indication of how activities can be adapted to use in different contexts. For example, a customer service survey could be used in a range of different situations, and an example involving the cost of supporting a football team would be equally suitable for a team in any sport.

Every care has been taken to ensure that the activities do not discriminate against any group of learners because of differences in gender or cultural background.
4.1 Functional mathematics in GCSEs

Most GCSE subjects have opportunities to develop functional mathematics. If we identify these opportunities and make the most of them, mathematics becomes a contextualised part of the whole learning experience and not just something that is ‘done in maths lessons’.

Applied art and design – working to project briefs

In this unit, learners are required to meet a project brief for a client. The specification states that this must take into account the constraints of cost and time. Mathematical techniques are clearly involved here and can be addressed in a way that emphasises the financial and time implications of meeting a customer’s requirements.

Applied business – business finance

Preparation for this unit will require learners to develop a whole range of mathematical techniques associated with finance and as such cover different types of calculations and the use of formulae at levels 1 and 2. There are also excellent opportunities to develop the use of IT for accounts and financial forecasting.

If the problems are set in context, learners will have opportunities to develop the functional mathematics process skills as well as the mathematical techniques that they need to demonstrate both for the unit and for their functional skills.

Suitable activities would include:

- covering costs of a new product or service
- creating a cash flow forecast
- creating a budget
- calculating the break-even point
- calculating profit and loss.

Applied ICT

Learners will have opportunities to develop some mathematical techniques when they study the use of spreadsheets in units 1 and 2, carrying out simple calculations and producing graphs and charts. This can be taken further if they study how spreadsheets are used in a business context.

Applied science

There are opportunities to develop functional mathematics techniques and processes in all the applied science units, but the unit ‘Science at work’ appears to be particularly rich. The portfolio of evidence is an ideal place to
demonstrate functional mathematics skills at either Level 1 or 2, depending on the individual learners and their application of the process skills and coverage.

The portfolio investigations will develop mathematical techniques that involve extracting and interpreting information, and applying the four operations to solving problems associated with yield, mass, cost, force and work efficiency. They may involve the use of formulae, and manipulating statistical information. There are also opportunities to check the accuracy of the results and present the information in appropriate ways.

**Business**

Different awarding bodies offer different units, but many topics lend themselves to developing functional mathematics, including the following.

**Accounting and finance**

Learners can develop and use their skills of representing, analysing and interpreting information in activities with budgets, cash flow forecasts, costs, break-even analysis and final accounts. They are also asked to calculate and interpret ratios to assess business performance. Any activity in these fields is likely to provide opportunities to develop functional mathematics process skills and techniques such as carrying out calculations using decimals in practical contexts, using formulae and interpreting results.

**Marketing**

There may be opportunities in this module to plan and carry out market research for a company or product. This type of activity will develop the process skills and the techniques of data handling to collect, represent and interpret data and to use statistical methods to investigate situations. It is also likely to involve the use of ICT.

**Design and technology – designing and making**

The options in design and technology offer a variety of opportunities to develop and practise functional mathematics processes skills and techniques. The ‘Designing and making’ component offers the best opportunity to follow through the process skills and apply the appropriate mathematical techniques as learners are required to carry out product analysis, model their findings, use a range of mathematical techniques and select effective methods of presentation, including the use of ICT as appropriate.

**Engineering – design and graphical communication**

This requires learners to develop a solution from a client design brief.

The specification requires the following elements:

- analysing client design briefs
- developing design specifications and solutions
• applying scientific principles
• producing and reading engineering drawings
• selecting appropriate drawing techniques
• communicating a design solution
• presenting the portfolio and prototype to the client.

To develop design ideas learners must be able to use the techniques of:
• research and analysis of information and data
• generation of ideas and solutions
• evaluation of ideas, solutions, testing and subsequent modifications
• two-dimensional and three-dimensional drawing and sketching techniques
• modelling techniques.

The GCSE engineering website provides a wealth of information and a number of assignments to support this unit. See www.gcseinengineering.com.

All these assignments, and any other task that meets the requirements of the unit, will provide an ideal model to develop and evidence functional mathematics at Level 1 or Level 2. The elements relate to the process skills and the mathematical techniques will be appropriate to the coverage/range. It is likely that an assignment that meets GCSE grade A*-C will be suitable for Level 2 functional mathematics.

Geography

The geography curriculum is a rich source of opportunities to develop and practise functional mathematics. The study of physical, human and economic geography involves learners in research, processing data and presenting their findings. Broadly speaking, the coursework projects are graded on the process skills of functional mathematics and include:
• collecting and selecting primary and secondary data
• representing data
• analysing and interpreting findings
• drawing conclusions.

The study of population, settlement, economic activities and energy are a few of the appropriate examples. Learners will use all aspects of the functional mathematics techniques, including statistical analysis, different types of calculation, and a variety of different methods of presenting their findings, including the use of ICT, as identified in the coverage/range statements in the functional mathematics standards.
Health and social care – promoting health and well-being
When learners carry out their investigations for this unit, they are likely to develop a variety of mathematical techniques associated with interpreting measurements such as blood pressure, peak flow, body mass index and pulse readings using graphical and other formats. They will probably carry out calculations involving weight and height, pulse rate and diet, use a formula to calculate body mass index and use ratios of, for example, smokers to non-smokers in a particular group of people. They may do some statistical analysis, perhaps comparing data from a group of people with the national statistics on obesity or incidence of heart disease. They will present their findings as part of their final report.

ICT
Coursework projects involving the use of spreadsheet software are likely to provide opportunities for learners to develop and practise their functional mathematics process skills and techniques. They may involve planning, making initial models of situations and deciding on methods and mathematical information to use. The processing and analysis of the data and interpretation and communication of results will include the use of ICT to carry out calculations and present findings.

Leisure and tourism – marketing in leisure and tourism
Although it is not required, learners may choose to carry out some research, compare their findings with marketing information, and present the result in a statistical format. This would give learners opportunities to develop their functional mathematics process skills and techniques. They may also use ratios to compare the marketing mix of their sample with the ideal mix. This may develop skills and techniques at Level 1 or 2.

Manufacturing
GCSE manufacturing is rich in opportunities to develop functional mathematics process skills and techniques at Levels 1 and 2 through:

- making sense of situations and representing them
- processing and using mathematics
- interpreting and communicating the results of analyses.

Activities are set in both familiar and unfamiliar contexts, the calculations are set at an appropriate level of difficulty, and learners are required to select and adapt their own models to arrive at a solution. There are opportunities to develop the full range of mathematical techniques at Level 1 or Level 2 as appropriate to the learner and the context.
Unit 1 – Designing products for manufacture
Learners are required to consider production details and constraints, properties of materials, scales of production and costs as part of developing a design solution. They will carry out a range of calculations associated with raw materials or ingredients, labour costs, product dimensions, tolerances, scale models, quality control, probabilities and use of formulae. They will present the results as part of a design solution.

Unit 2 – Manufactured products
In this unit learners will interpret technical information about components, ingredients, materials, etc. from a production plan, calculate and maintain levels of resources, identify correct calibrations on machines, work with dimensions, quality control, probabilities and formulae, and present the findings in a production schedule.

Unit 3 – Application of technology
Unit 3 involves investigating a manufactured product and the impact of new technology, interpreting technical information, carrying out a range of calculations involving dimensions, quantities and scale, using formulae and statistics about, for example, market share, range of products and energy consumption.

The GCSE manufacturing website offers a wealth of suggestions and teacher support materials. See www.gcseinmanufacturing.com

Mathematics
Learners will usually develop their functional mathematics techniques in specialist mathematics lessons but they will also need opportunities to develop all their functional ‘process’ skills. If the problems that learners tackle in mathematics lessons are limited to the development of mathematical techniques (the analysis part of the process), they will have little opportunity to use the representing and interpretation skills that are also core to functional mathematics.

Learners will have opportunities to develop and practise all the functional mathematics process skills if they are given more open-ended activities that require a problem solving approach such as that described in section 2.

Physical education
Learners will have opportunities to develop and practise their functional mathematics process skills and techniques as part of their physical education course. They may be required to collect primary and secondary data, analyse performance, including making comparisons against benchmark data, and present their findings using ICT as appropriate.
Science
There will be opportunities for learners to develop and practise their functional mathematics skills throughout their GCSE science programme. They are likely to have opportunities to plan and carry out experiments and analyse and present the results, do research and compare results to benchmark data, using formulae to calculate results. Presenting learners with open-ended tasks will help them to develop their functional mathematics process skills alongside their scientific knowledge and techniques.

4.2 Functional mathematics in work experience

Analysing how time is spent
This activity could be carried out on work experience or applied to full-time, part-time or holiday jobs. It gives learners the opportunity to develop the three process skills, using their mathematical techniques at the appropriate level.

How did I spend my time?
Your task is to do an analysis of how you spend your time during work experience. You should include all the activities you carry out as part of your work, including breaks and the different jobs that you have done each day.

You will need to decide how you will record the information, what calculations you will do and how you will present the findings.

The task appears straightforward at first glance, but the learner will have to make a number of decisions about collecting the information, such as what they will need to record and how they will record it.

They have been asked to ‘analyse’ their time, which should make them think about using statistical methods. At levels 1 and 2 they could be using a range of statistical techniques to analyse and compare their use of time. Level 2 responses would include justification as to why they have used particular techniques.

Learners are required to present their findings in a way that is suitable for the audience, which might consist of fellow-learners, a group of teachers, managers at the workplace, or any other group that you nominate. The findings might be presented as a written report or as an oral presentation with visual aids. Learners may use ICT for both the calculations and the presentation, and will possibly use appropriate graphics, such as pie charts. This will enable them to practise their functional ICT skills.

The activity also gives learners an opportunity to practise and develop their functional English skills, perhaps by giving a presentation (speaking and listening) or in a written report.
Adapting the activity

This activity is appropriate to learners at different levels from any cultural background and in any vocational context that includes recording time. For example, at Entry 3 learners may keep a record of appointments in a diary at a hairdressing salon, making sure that they leave enough time for each type of treatment and that there are not too many clients in at any one time.

Learners can be given more support at the lower levels, for example by providing templates, additional information or more structured guidance. This will enable learners to approach challenging problem-centred activities with more confidence.

Changing the level

Entry 2
Learners write down the time they start and finish activities on a recording sheet you have provided.

Entry 3
Learners estimate the time it will take to carry out a task (in minutes). They use a diary or other recording sheet to write down the start and finish time and check the time taken against the estimate. They compare times taken to do different tasks.

Level 1
Learners at this level may need some guidance, but the final decisions about how they carry out the task should be their own. These decisions include what to record of their daily activities and how to do this, what calculations they will use and how they will present their findings. They may not carry out a wide range of different tasks. The analysis may include calculations of mean and range for their different activities and the use of appropriate charts to draw some straightforward conclusions about how they have used their time.
Customer service
This activity requires learners to plan and carry out a survey about the canteen or restaurant in their school, college, training centre or place of work.

Your opinion matters!
Some of your friends have been complaining about the food and service in the canteen. They say that everyone is fed up, the food costs too much and the service is poor. Are they right?

Find out what other people think about the canteen. Plan how you will do this, analyse your data, and present your findings to the canteen manager.

Here are some things for you to consider:
- You may want to ask about prices, menus, cleanliness and service in general.
- Will you give people a questionnaire to fill in or will you interview individuals and record their answers?
- What calculations will you need to do?
- How will you present your results?
- Will you need to use IT facilities?

This is presented as a Level 1 activity with a list of hints to help the learners to plan and carry out the survey. The coverage/range of mathematical techniques at this level includes collecting and recording discrete data and finding the mean and range.

Adapting the activity
Data handling techniques are included in the coverage/range at levels 1 and 2 although the process skills required may be higher or lower than this. Practising the process skills for this type of activity will help learners to develop their skills in real contexts.

A customer satisfaction survey can be adapted to use in any café, restaurant or canteen in a garden centre, shop, leisure centre, retirement home, etc. It could also be adapted for other service industries, such as a survey of customer satisfaction in a hairdressing salon (including questions about prices, treatments, cleanliness, courtesy), estate agent (courtesy, accuracy of house details), garage (customer care, speed of service).
### Changing the level

**Level 2**

For Level 2 learners it would be more appropriate to omit the advice and suggestions. This would give learners more scope to develop their process skills at the appropriate level.

At Level 2 you would expect a wider range of data handling techniques, and justifications for the methods used and the results. Learners may decide to compare customer numbers and waiting times at peak and off-peak periods, the range of food available at different times of the day, average spend per customer, and other relevant information. They may also decide to interview the canteen manager to find out the other side of the story – costs, staffing, etc. – which would show that they are representing and analysing the situation more effectively. You would also expect the report to show signs of clear thinking in terms of interpreting the findings and appropriate use of the mathematical analysis.

**Mixed level groups**

You could present the task to all learners in the group without the list of hints. Those who need more guidance to demonstrate their process skills at Level 1 could then be given the extra information to help them with their planning.

### 4.3 Functional mathematics in the workplace

Most jobs include activities that can be used to develop and practise functional mathematics processes and techniques. In catering, hairdressing, horticulture and care, for example, there are situations where learners are required to plan and carry out activities that involve weighing and measuring, using ratio to mix colours or chemicals and scaling quantities up and down. Measuring areas, quantities and volumes is fundamental to the construction industry. Calculations involving money are essential to retail and service industries.

**Monitoring stock**

All retail businesses have to monitor sales, stock levels and wastage. The activity could be to study how the process of stock control is carried out in a particular work environment or to develop a stock control system for a business. At Entry level, learners may help with stock-taking and plan how they will count and record the stock. They may be asked to fill in their findings on a standard form. Learners at Level 1 may be asked to calculate the value of stock in a section of the stores. This activity could be adapted to any context that involves control of stores, for example residential care, or the hospitality and catering industry, looking at packaging, storage, wastage, costs, etc. in a restaurant, sandwich shop or canteen.
Measuring up or costing a job
Measuring up a job, working out the materials needed and working out the cost is an essential task in all areas of the construction industry and can be adapted to many other contexts, for example planning hard landscaping or planting in horticulture, designing and costing a set for a drama production.

Design and layout
Planning the layout of a reception area in a hairdresser or other business, a play area in a nursery, the day room in a care home or day centre, or car parking, all involve measuring, calculating and considering a range of options. This could be extended to taking into account the cost of a number of alternatives.

Planning a route
In the context of a business that makes deliveries, the activity will involve using maps to plan a route, with mileage and approximate timings for the van or lorry driver, to minimise journey times and mileage and inform customers of approximate delivery times. It could be adapted to any context where someone has to travel from place to place during their working day, such as contexts for health visitors, TV repair engineers or couriers.

4.4 Functional mathematics in citizenship
There are many opportunities to apply functional mathematics in citizenship. Surveys and investigations into local and national crime figures, central government and local council spending, and issues such as sustainability, recycling, local business trends and many more can be used to embed mathematical processes and skills.

Crime statistics
This activity will allow learners to practise their research skills to find information at both national and local level and relate their findings to their own experience.
Teaching and learning functional mathematics: 4. Cross-curricular activities

How likely am I to be a victim?
This activity involves a study of local and national crime statistics. The local and national press and TV are full of stories about violent crime. They sometimes give the impression that it is not safe to walk the streets.
But what is the real situation? Are things really that bad? Are they worse in some areas than others? Are some groups of people more at risk than others?
What do the official statistics of crime tell you?

Setting the level
In this case, the level has been left open so that the activity can be used at a range of different levels.

Level 2
Learners would probably be able to approach this activity with very little guidance, especially if they have well-developed process skills. They may search the crime statistics on the internet, for example starting with www.crimestatistics.org.uk, and compare the local and national figures. They may also make a study of the local events by studying back issues of the local paper.

Level 1
At this level it may be necessary to give learners some help with planning their activity, pointers to finding the information they may need and more support as they carry out the process and complete their analysis of the situation.

Entry level
Learners may be given some simple statistics, perhaps in the form of a table, pick out the relevant information, and draw some conclusions.
Council spending
This is another activity that could be used as part of a citizenship programme as well as an opportunity to practise processes and techniques in functional mathematics. It is an investigation of how the local council raises money and what the council spends it on. It will give opportunities to use IT to search for information, carry out calculations and present results in a variety of different ways.

So what do they do with our council tax anyway?
Have you ever wondered what council tax is spent on? Now is your chance to find out.
Check out the range of local services that the council provides in your area and see what you can find out about how they are funded and where the money comes from.

Adapting the activity
It would be possible to adapt this activity to investigate other forms of public funding, such as health spending or, on a smaller scale, the funding for a school or college. Alternatively learners could look at the finances of a charity, for example how the income is raised and how it is spent.

Changing the level
This is another open-ended activity which could be used by learners at different levels and by mixed-ability groups.

Level 2 The teacher presents the activity as given here.
Level 1 Learners may need more support – perhaps a list of hints and some ideas to get them started.
Entry level Learners could be given a council tax bill and perhaps the local council information sheet or newsletter to find out as much as they can about how much money is raised, where it comes from and how it is spent. The amount of information and support given will depend on the ability level, but learners should be given opportunities to practise the process skills as well as the performance and coverage/range for their level.

In the news
This activity focuses on some issue of global importance, such as global warming or child poverty. Learners would be given some stimulus, for example a newspaper article, a podcast, an item from the BBC news website or a recording of an item on the national or local news. Their task would be to find appropriate statistics and compare their findings to the news items.
may be useful as a group activity, where learners compare the results of searching for information from different sources.

4.5 Functional mathematics in personal finance

Financial matters are of interest to all learners of all ages at all levels. From the simple ‘Have I got enough money to buy a cup of tea and a bun?’, to the slightly more difficult ‘If I catch the bus home and have chips at lunchtime, will I have enough money left to go to see a film this evening?’, to ‘Can I really afford to buy and run a car or should I get a motorbike?’

Several financial institutions have websites and resources that can be used for financial literacy topics and there are opportunities to develop and practise the processes and techniques that learners need to develop their financial awareness. An internet search on ‘Financial literacy’ will bring up a number of sites to check, for example:

‘NatWest Face2Face with Finance’ on www.natwestf2f.com is designed for use by learners aged from 11 to 18 years.

‘Money matters to me’ is a website developed by NIACE and supported by Prudential plc that focuses on family finances. See www.moneymatterstome.co.uk

‘Support for learning, Financial education’ provides dozens of links to websites about all aspects of personal finance. See www.support4learning.org.uk/money/financial_education.cfm

The following activities address spending, borrowing, saving and investing.

**Spending**

The following activity may be useful for Entry level learners to develop and practise their functional mathematics process skills and techniques. It is based on a shopping activity and focuses on finding the best value for money.
What is the best buy?
When you go shopping for food, do you look out for bargains? Next time you go to the supermarket or local shop, see how many things on your shopping list are on special offer and check how much you could save on the normal price. Try to find different types of reductions and 'best buys'.

Try to answer the following questions:

• How much have I spent?
• How much have I saved?
• Are ‘offers’ always cheaper?

Which reduced items did you find that you didn’t buy? Why didn’t you buy them?

This activity has some guidance to support Entry level learners. It may be useful to start the work with a discussion about when and why goods are reduced, for example when they are near their sell-by date or because they are not popular. Other points to discuss may be the cost of large sizes – are they always cheaper? If buying a large size of a food product, can you use it all before it is past its best?

There are some useful materials, for example in the Skills for Life materials for embedded learning – social care, module 4 ‘Figure it out’ which may be useful for learners to practise the mathematical techniques and help with discussions in preparation for the activity. See www.dcsf.gov.uk/readwriteplus/embeddedlearning/

Adapting the activity
Although aimed at personal shopping in a supermarket, the activity could be adapted for a range of contexts, including the social care example mentioned above.

Changing the level

Level 2 Learners could be given a budget and a scenario such as catering for a large party at the end of a course, an important birthday, or a family celebration. If they are in work, an opportunity may arise for a leaving ‘do’ or retirement party.

Level 1 As Level 2, with more guidance.
Teaching and learning functional mathematics: 4. Cross-curricular activities

Borrowing

Can I afford this?

Store cards can be a tempting way to secure a discount on your shopping but shoppers often end up paying extra through high interest charges.

Imagine that you’re at the checkout, weighed down with purchases, and the assistant at the till offers you a 10% discount on everything – if only you’ll sign up to use the shop’s handy store card.

Are you really being offered something for nothing, a nice discount and an opportunity to buy now and pay later? What is the catch?

Here are some things to think about.

- Research different store card charges and then compare the alternatives, such as paying cash, using a credit card or getting a bank loan.
- Base your calculations on the cost of buying something that you really want – perhaps a new outfit, a flat-screen TV with DVD player, a bike or something else.

This activity has been written for Level 1 learners, hence the ‘Here are some things to think about’ section. The activity still allows considerable scope for applying the process skills and selecting appropriate mathematical techniques.

Adapting the activity

The activity could be adapted to look at other methods of personal borrowing, with the emphasis on credit cards, bank loans, finance deals to buy a car, or ‘buy now, pay later’ schemes that are popular with electrical and furniture retailers.

This could also become a group activity. There are other ways of borrowing money and perhaps learners could investigate as many schemes as they can find, looking at the ‘pros and cons’ such as interest rates, secured and unsecured loans, credit ratings, maximum and minimum amounts for advances, etc, and ranking them based on a set of criteria that they have decided as a group. An investigation of this kind would be open-ended and decided by the group, with each individual playing their part. It could be appropriate to include mixed-ability learners in the group.
Changing the level

Level 2  Remove the ‘Here are some things to think about’ section to give learners control of the process and the mathematical techniques that they will use.

Level 1  Use as provided.

Entry level  Learners can investigate leaflets or other advertising for store cards and credit cards. They could use examples of store card and credit card bills and identify the purchases, amount of interest and the interest rate.

Savings

Learners can compare different types of savings, including National Savings, premium bonds, deposit accounts, ISAs, etc. This may involve looking at interest rates, regular savings, minimum balances and loss of interest for withdrawals. Learners could also consider aspects such as how safe the money is.

Investments

This activity looks at investing money.

Groups of learners investigate different types of investment, develop and track their investment portfolio, and keep within a set budget. The activity could be set up as a competition with small groups selecting different types of companies and investments to compare performance over a set period.

This type of activity is suited to learners on business courses or those with an interest in investments and the stock market. It provides an excellent opportunity to develop Level 2 functional mathematics process skills in a realistic though simulated activity which is carried out over a period of time.

For those who want to take this activity further, the ‘ifs School of Finance’ includes the ‘Student Investor Challenge’ and ‘Uni Investor Challenge’. See www.ifslearning.com/financial_capability.
4.6 Functional mathematics in hobbies and interests

Most learners will be able to identify with activities associated with their hobbies and interests and may be more willing to work with such a topic, rather than following a set task. There are functional mathematics implications in most hobbies. Decisions associated with costs of materials, equipment, travelling, membership fees, etc are often important when deciding to take on a new hobby.

Following sport

<table>
<thead>
<tr>
<th>Are they worth it? The cost of being a supporter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you support a particular football team? Do you go to home games? Do you follow your team to away games? Do you buy the team’s merchandise?</td>
</tr>
<tr>
<td>Have you thought about the total cost of supporting your team for a whole season?</td>
</tr>
</tbody>
</table>

This very open-ended activity gives learners an opportunity to work out exactly how much they spend on supporting their team.

Adapting the activity

The activity could be adapted to look at following other sports, perhaps motorbike racing, ice hockey or athletics. It may be interesting to consider the cost of following a favourite band. In a wider context, all hobbies have cost implications such as travel expenses, equipment, materials, special clothing, entrance fees, membership fees, etc. Learners may like to look at the cost of taking up a new hobby or pastime.

Changing the level

<table>
<thead>
<tr>
<th>Level 2</th>
<th>As it stands, this activity would give Level 2 learners scope to develop and practise their process skills and techniques for functional mathematics. They can compare the cost of different options for travelling, buying a season ticket, joining a supporters’ club, etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Learners may need some extra guidance to get started on this activity.</td>
</tr>
<tr>
<td>Entry level</td>
<td>Learners can investigate prices of tickets and work out the cost of going to all the home games. They could look at travelling to a particular away game and sort out times of trains or buses and the cost of tickets.</td>
</tr>
</tbody>
</table>
Cooking a meal for friends
This would involve planning a menu, working out quantities for recipes (scaling up or down), and associated costs. The activity could be adapted for different levels of learners. Starting with a recipe for a main course for four people and a price list for a range of desserts, an Entry level learner could be asked to plan a meal for two people and find some prices. For Level 1 or Level 2 learners, this may entail being given a budget, recipes and other information and asked to plan a larger event, maybe for nine or ten people.

Downloading music
Learners may be interested in a technical investigation associated with speed, bandwidth and memory size.
HMP Wormwood Scrubs

In 2007/08, about 100 learners have piloted functional mathematics. The learning was delivered in ‘real time’ because the learners were serving short sentences of between two weeks and three months. They were asked what they needed to learn to ‘turn their lives around’, and the functional mathematics was tailored to respond to their answers, to make it relevant, practical and real.

The learners said that if they had been taught mathematics in this way at school, they perhaps would not now be in prison. They highlighted aspects of money management and budgeting, which allowed the ‘maths’ to be pulled out of the tasks, as particularly beneficial.

There was 100% achievement in mathematics at Level 1, and 85-90% at Level 2.

Staff had at first been reluctant to support the pilot as they found change difficult. However, external training was arranged and take-up was good. This had a positive effect on attitudes.

The FSSP regional network meetings were very useful in terms of forward planning and keeping abreast of developments. The GCSE reforms were also motivating because of the mandatory functional skills element.
5. Assessment, progression and mastery

Contents

5.1 Assessment relates to the process skills
5.2 Assessment and progression
5.3 Knowing when a learner has achieved mastery
5.4 Exemplification of evidence at Levels 1 and 2

This section will help practitioners to assess and support their learners as they become more confident and make progress in functional mathematics. It is intended to support teachers in using assessment for learning in relation to functional mathematics.

The section has four key messages.

- Assessment in functional mathematics should relate to the process skills set out in the standards – representing, analysing, interpreting.
- Learners may have spiky profiles in these process skills, ie they may have made more progress in relation to one or two of the skills than in the other(s).
- Progression in relation to the process skills relates to the degree of challenge of the problems solved – this includes their complexity, their familiarity, the difficulty of the mathematical techniques employed, and the degree of independence given to learners.
- Judging when learners have achieved a level involves assessing their work in relation to the performance statements and the coverage/range statements in the standards.

5.1 Assessment relates to the process skills

Assessments of progress in functional mathematics are about the extent to which learners have managed to:

- make sense of situations and represent them using mathematics
- analyse situations by processing the mathematics
- interpret and communicate the results of the analysis.

The standards set out these skills in a table with a column for each skill, as shown in Figure 5.1 (overleaf). Bullet points listed under each skill expand on the headings. These bullets are staging posts on learners’ journeys as they solve problems using mathematics.
Figure 5.1

The process skills are set out in a table in which there is a column for each skill.

<table>
<thead>
<tr>
<th>Representing</th>
<th>Analysing</th>
<th>Interpreting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Making sense of situations and representing them</td>
<td>Processing and using mathematics</td>
<td>Interpreting and communicating the results of the analysis</td>
</tr>
<tr>
<td>A learner can:</td>
<td>A learner can:</td>
<td>A learner can:</td>
</tr>
<tr>
<td>• recognise that a situation has aspects that can be represented using mathematics</td>
<td>• use appropriate mathematical procedures</td>
<td>• interpret results and solutions</td>
</tr>
<tr>
<td>• make an initial model of a situation using suitable forms of representation</td>
<td>• examine patterns and relationships</td>
<td>• draw conclusions in light of the situation</td>
</tr>
<tr>
<td>• decide on the methods, operations and tools, including ICT, to use in a situation</td>
<td>• change values and assumptions or adjust relationships to see the effects on answers in the model</td>
<td>• consider the appropriateness and accuracy of the results and conclusions</td>
</tr>
<tr>
<td>• select the mathematical information to use.</td>
<td>• find results and solutions.</td>
<td>• choose appropriate language and forms of presentation to communicate results and conclusions.</td>
</tr>
</tbody>
</table>

In each column, there are bullet points that expand on the headings.

It is important to regard these bullets as staging posts on learners’ journeys as they solve problems using mathematics; they are not separate objectives that learners can address in isolation.
The process skills are therefore parts of a single overall process – solving problems using mathematics. In carrying through this process, most of the skills represented by the individual bullets will need to be used every time; often, all the skills will be needed.

Assessing the process skills

Being functional with mathematics means being able to use these process skills to solve problems. It follows that assessing progress towards becoming functional with mathematics means making judgements about the extent to which a learner is able to use the process skills. How well the learner’s response answers the real-world problem is more important than the mathematical techniques employed. That said, a key question for teachers is: ‘How should I make these assessment judgements?’

The answer is that you should make judgements about performance in the three process skills – representing, analysing and interpreting – in terms of the individual bullet points listed under that skill. The purpose of these assessments is to inform the next steps in your teaching; for example you may decide that more activities requiring representing or interpreting skills are needed.

There is no need to assign a level to learners’ performance in each of the three skills. Moreover, you should not need to make an overall judgement that a learner’s work in functional mathematics is at a particular level. This summative assessment will be made in tasks set by an awarding body at the end of the course.

However, you may wish to make records based on your professional judgements. Such records of learners’ performance should be kept to a sensible minimum.

Learners may have spiky profiles in process skills

Sometimes a learner’s performance in representing, analysing and interpreting will be at a different level for each skill. Indeed, it is quite likely that learners will demonstrate different levels of performance across the three process skills, as they describe quite different capabilities. For example, a learner might demonstrate high-level representing and interpreting skills, but be much less capable at analysing. Similarly, a learner’s spiky profile is likely to vary according to the context of the problem or task, for example they will be more confident, and hence show more independence, in some contexts than in others.

Showing different levels of performance in relation to these key aspects of assessment is what is meant by a learner having a spiky profile.
5.2 Assessment and progression

Because the basis of assessment is the same at all levels, assessment judgements are made based on the extent to which learners have demonstrated success in using the process skills. Within the process skills themselves, there is no ladder of achievement that indicates how to recognise the steps learners make as they progress – all the process skills can be used at a low level, if the problem is simple enough.

However, while the process skills are specified in a way that is appropriate for all levels, the standards do differentiate between:

- problems at different levels
- learners' performance at different levels
- the mathematical techniques likely to be used at each level.

The standards list ways in which problems that can be solved using mathematics can differ in their demands on learners, depending on:

- their complexity
- their familiarity to the learner
- their technical (mathematical) demand
- the degree of independence given to the learner.

These four dimensions of difficulty impact on the overall demand of a task.

In many mathematics lessons, the focus is on mathematical techniques; the technical demand of the techniques determines the difficulty of the work. In functional mathematics, by contrast, a more demanding task is one that, overall, has a greater demand in terms of all four dimensions. A task in functional mathematics must therefore include:

- a degree of complexity
- low familiarity
- a degree of independence required to solve it.

These requirements are expressed in the performance statements.

Using the performance statements to support assessment

The performance statements in the standards describe what teachers may expect of learners in relation to the issues of the complexity and familiarity of the task, and learner independence, at each level. For example, see Figure 5.2: at Entry 3, complexity is addressed in the performance statement 'learners can... obtain answers to... simple given practical problems that are clear and routine'. At Level 1 similar requirements apply to 'practical problems' in general. By Level 1, therefore, learners will have moved beyond problems that are 'simple', 'clear' and 'routine'.
However, different balances of complexity, familiarity and learner independence can produce tasks that allow responses at any level. For example, a task with Level 2 complexity, or that demands a fuller analysis of the problem, may use only Level 1 mathematical techniques. Such a task might be equivalent in demand to a familiar problem that requires straightforward uses of Level 2 mathematical techniques.

**Figure 5.2**

**Entry 3**

**Performance**

Learners can:
- understand practical problems in familiar and accessible contexts and situations
- begin to develop own strategies for solving simple problems
- select and apply mathematics to obtain answers to simple given practical problems that are clear and routine
- interpret and communicate solutions to practical problems in familiar contexts and situations
- use simple checking procedures.

At Entry 3, complexity is addressed in the performance statement – ‘Learners can… obtain answers… to simple given practical problems that are clear and routine’

**Level 1**

**Performance**

Learners can:
- understand practical problems in familiar and unfamiliar contexts and situations, some of which are non-routine
- identify and obtain necessary information to tackle the problem
- select and apply mathematics in an organised way to find solutions to practical problems for different purposes
- use appropriate checking procedures at each stage
- interpret and communicate solutions to practical problems drawing simple conclusions and giving explanations.

At Level 1, similar requirements apply to 'practical problems' in general. By Level 1, therefore, learners will have moved beyond problems that are 'simple', 'clear' and 'routine'.
Similarly, the learner’s independence is relevant in, for example, the Entry 1 requirement that ‘learners can… use given methods’, while at Entry 3 ‘learners can… begin to develop own strategies’ and, at Level 2, ‘learners can identify the… problem and the mathematical methods needed to tackle it’. Here too, the performance statements indicate a ladder of progression that learners are expected to move up. For functionality to relate to the real world, a substantial proportion of the assessment tasks should be non-routine, ie not imitations of something the learner has been shown how to solve. Thinking through a problem with unfamiliar aspects is essential.

Familiarity is also mentioned at, for example, Entry 3 where ‘learners can… understand practical problems in familiar… contexts…’ and at Level 1, where ‘learners can… understand practical problems in familiar and unfamiliar… contexts…’.

The implication for assessment is clear – learners can be regarded as performing at Level 1 only after they have solved at least some problems in unfamiliar contexts.

**Using the coverage/range statements to support assessment**

The coverage/range statements describe the technical demand of appropriate problems at a level. They indicate the mathematical skills and techniques that are likely to be used by learners performing at that level.

It is important to note that the relevant skills are not set out in full. The content that is listed should be regarded as equivalent to and standing for content at particular levels of the National Curriculum, the related adult numeracy standard, and the related application of number standard (at Levels 1 and 2 only).

The mathematical techniques at each level set out the mathematics that could be used by learners, or that teachers who are setting problems can assume to be available to learners. It is not appropriate to require any particular technique to be used in solving a problem; as process skills improve, a problem might be solved using less demanding mathematics. For example, a learner at Level 2 may deal with a problem that involves calculating a price including VAT at 17.5%, by first calculating the price, then finding 17.5% of this amount, and finally adding the two quantities. This does not demonstrate the full Level 2 appreciation that multiplying the price by 1.175 is the simplest method.

Teachers should note that almost all the techniques listed in the coverage/range statements in the standards apply to the analysing aspect of the process skills. It follows that these statements give little help in supporting judgements about the representing and interpreting aspects.
5.3 Knowing when a learner has achieved mastery

A learner has achieved mastery of a level in functional mathematics when that learner demonstrates the ability to use the process skills in ways that satisfy the performance and the coverage/range statements at that level. As already indicated, the performance statements illustrate the challenge of problems at the level; this challenge must be fully satisfied for the level to be achieved. The coverage/range statements indicate the difficulty of the mathematical techniques that can be expected; a level may be achieved if the coverage/range statements for the level are a better match with the mathematical techniques demonstrated by the learner than those of any other level.

In practice, this means, for example, that a learner working towards Entry 3 must clearly demonstrate success in tasks that are of the difficulty indicated by the five performance statements in the Entry 3 standard. On the other hand, the same learner, working towards Entry 3, need not demonstrate all the coverage/range statements in the Entry 3 standard. Rather, it is sufficient for this learner to demonstrate enough of the Entry 3 coverage/range statements (or mathematical techniques of equivalent difficulty from the National Curriculum or adult numeracy standards) to make them a better description of the techniques used in the learner’s work than the coverage/range statements at the levels above or below Entry 3.

The soundest measure of the level of complexity of a task is the difficulty it presents to learners, when compared with straightforward exercises in mathematical techniques. As teachers become more familiar with functional mathematics, they will develop a capacity to recognise the level of difficulty of tasks in functional mathematics and to compare this with the difficulty of the standard mathematical exercises with which they are more familiar.

Bearing these two scales of difficulty in mind, it follows that, if a complex ‘functional’ task is of the same level of difficulty as a ‘standard’ Entry 2 exercise on mathematical content, it is Entry 2 for functional mathematics.

5.4 Exemplification of evidence at Levels 1 and 2

We give below some exemplification of learners’ work at Levels 1 and 2. The two learners have tackled the same problem – investigating the use of mobile phones.
Level 1 – Learner A

Problem: Investigate the use of mobile phones

I am going to ask 30 people about their mobile phones. I am going to ask my friends and family. I think that most people will own a mobile phone. I think that people will use it most to send texts.

The questions I am going to ask are:

- Do you have a mobile phone?
- Is it on a pay-as-you-go or monthly rental scheme?
- How many texts do you send each day?
- How many phone calls do you make each day?

I will collect my data using tally charts and then show my results on charts and use averages.

Results

1. Yes 26
   No  4

2. Pay-as-you-go 11
   Rental 15

3. Texts:

   |   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
---|---|---|---|---|---|---|---|---|---|---|
   | 3 | 0 | 0 | 1 | 0 | 2 | 1 | 0 | 4 | 0 |
   | 10| 11| 12| 13| 14| 15| 16| 17| 18| 19|
   | 8 | 0 | 3 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
   | 20| 21| 22| 23| 24| 25|
   | 2 | 0 | 0 | 0 | 0 | 1 |

4. Calls:

   |   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
---|---|---|---|---|---|---|---|---|---|---|---|
   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
   | 1 | 0 | 5 | 0 | 4 | 4 | 3 | 3 | 2 | 0 |
   | 10| 4 |
I am going to show these results on bar charts.

These charts show that I was right and most people have a mobile phone. Most have monthly rental.
The bar charts on texts and calls are not very good. There are too many gaps so I am going to use pie charts instead.

10 was the most popular number of texts.
2 was the most popular number of calls. The range for calls was 10 and the range for texts was 25. So the range for texts was bigger.
I am going to work out some averages.

**Texts**

<table>
<thead>
<tr>
<th>Number</th>
<th>Count</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>0 × 3 = 0</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3 × 1 = 3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5 × 2 = 10</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6 × 1 = 6</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>8 × 4 = 32</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>10 × 8 = 80</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>12 × 3 = 36</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>15 × 1 = 15</td>
</tr>
<tr>
<td>20</td>
<td>2</td>
<td>20 × 2 = 40</td>
</tr>
<tr>
<td>25</td>
<td>1</td>
<td>25 × 1 = 25</td>
</tr>
</tbody>
</table>

Total: 26  247

Mean = 247 ÷ 26 = 9.5

**Calls**

<table>
<thead>
<tr>
<th>Number</th>
<th>Count</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0 × 1 = 0</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2 × 5 = 10</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4 × 4 = 16</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5 × 4 = 20</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>6 × 3 = 18</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>7 × 3 = 21</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>8 × 2 = 16</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
<td>10 × 4 = 40</td>
</tr>
</tbody>
</table>

Total: 26  141

Mean = 141 ÷ 26 = 5.4

The mean of the texts is bigger than the mean of the calls so I was right.

**Conclusion**

I have found that most people have mobile phones and they send more texts than calls. This might be because lots of people get free texts on their phones.

If I was to do this again I would improve it by asking more people.
Commentary on Learner A’s work

This solution is somewhere in the middle of Level 1. It uses Level 1 coverage and performance but, more particularly, the process skills are at that level. More of the same would not move the work towards Level 2. For Level 2 there needs to be a more complex approach.

To explain this judgement more fully, we set out below some of the evidence that teachers would normally assess ‘in their heads’. There is no need to keep records at this level of detail.

1. Representing

In this problem, representing is about the planning of the data collection: deciding what aspects to investigate, making hypotheses, writing the questionnaire, and deciding who to ask. In this piece of work the learner has made some hypotheses. They are relevant and appropriate. The learner has recognised that the problem can be represented using mathematics and has decided on the methods to use. In designing the questionnaire, the learner has made an initial model of the situation using suitable forms of representation. The plan demonstrates that the learner has selected the mathematical information to use. These points demonstrate that Level 1 has been achieved.

However, the hypotheses are separate statements and no attempt is made to link them. They also leave some important questions unanswered, for example: What age is the user? A more coherent approach, which sees the problem as a single problem rather than a series of mini-problems, would move the work towards Level 2.

A wide range of people may have been asked but the impression is given that the sampling was opportunistic only. To reach Level 2 some thought should be given to obtaining a representative sample.

For the questionnaire, grouped boxes for some of the responses would have been more appropriate. This would be expected for Level 2.

2. Analysing

In processing and using mathematics the learner has chosen bar charts to represent the data; this is consistent with working at Level 1. There was some awareness that the bar charts were not always the most appropriate representation and two were replaced by pie charts. This is moving towards the thinking expected at Level 2, but in this case the replacement did not improve the analysis very much and this was not commented on.

However, no explanation was given for the choices of charts. For Level 2 some discussion about the choices of charts and/or statistical techniques and their suitability would be expected.

The calculation of means was an appropriate mathematical procedure and the comparison was useful. The comparisons move the work towards Level 2.
In calculating the means, there was no realisation that people had probably estimated the number of calls and texts (thus explaining the frequency of 10 or 15 while never having an instance of 13). The data was accepted at face value without any real thought about its accuracy. Some realisation that the data was probably based on estimates would be expected for a Level 2 piece of work. This may appear by using grouped data boxes in the questionnaire or as part of the analysis.

3. Interpreting

In this solution, each chart and calculation has been interpreted and there is a simple comparison. There is a very basic commentary linking the work together. The overall results of the analysis are communicated briefly in the conclusion and there is a valid suggestion about the reason for the results. This is sufficient to achieve a Level 1.

To reach Level 2 the interpretations would need to be more detailed and linked together with more commentary. The conclusion should be written with reference to the data collection, for example: were there any reasons why the findings might not be valid? Could any bias have crept in due to sampling?

At Level 2 more detailed reasons for the results relating to a real-life context would be expected.
Teaching and learning functional mathematics: 5. Assessment, progression, mastery

Level 2 – Learner B

Problem: Investigate the use of mobile phones

I am going to ask 40 people about their mobile phones. I am going to ask my friends, family and other people that I know. I am going to make sure that I ask at least 5 from each of the age groups:

15–20; 21–30; 31–40; over 40

I think that most people will own a mobile phone and I think that the ones that do not will be over 40.

I think that people will use it most to send texts but I think that most of the texts will be from the people under 30. I think that people over 30 are more likely to use it for calls. This is because they have not grown up with a mobile phone and some of them do not know how to use one properly.

The questions I am going to ask are:

Do you have a mobile phone?

On average how many texts do you usually send each day? Is it:

0 1–5 6–10 11–20 over 20

How many phone calls do you usually make each day? Is it:

0 1–5 6–10 11–20 over 20

What age are you?


I am going to group my data for the number of texts and calls as I do not think people will be able to remember accurately exactly how many texts and calls they make and it might not be the same each day.
Results

1. Yes 32
   No 8

2. Texts:
   0 1–5 6–10 11–20 over 20
   4 7 13 6 2

3. Calls:
   0 1–5 6–10 11–20 over 20
   6 14 7 5 0

   15 9 2 3 4 7

Only 8 out of 40 did not have a phone. Of these 3 were over 40 and the other 5 were all in the 26–30 age group. I was surprised by this. I am going to show the texts and calls on bar charts so they can be compared.

Texts
These show that people make more texts than calls.

I am going to work out an estimate of the mean for each to see how big the difference is.

For texts:

<table>
<thead>
<tr>
<th>Calls</th>
<th>0</th>
<th>1–5</th>
<th>6–10</th>
<th>11–20</th>
<th>over 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midpoint</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>15.5</td>
<td>25</td>
</tr>
<tr>
<td>Totals</td>
<td>4</td>
<td>7</td>
<td>13</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>104</td>
<td>93</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

Mean = 268 ÷ 32 = 8.375

I used 25 for the midpoint of the last group because it seemed sensible.

For calls:

<table>
<thead>
<tr>
<th>Calls</th>
<th>0</th>
<th>1–5</th>
<th>6–10</th>
<th>11–20</th>
<th>over 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Midpoint</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>15.5</td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>6</td>
<td>14</td>
<td>7</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>42</td>
<td>56</td>
<td>77.5</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Mean = 175.5 ÷ 32 = 5.484

On average people make about 3 more texts than they do calls.

I want to know if there is a difference between old and young so I am going to split the data. I will look at the age groups for 30 and under and for over 30.
### 30 and under:

<table>
<thead>
<tr>
<th></th>
<th>0 texts</th>
<th>1-5 texts</th>
<th>6-10 texts</th>
<th>11-20 texts</th>
<th>over 20 texts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Texts</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>Calls</td>
<td>0</td>
<td>3</td>
<td>11</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

### Over 30:

<table>
<thead>
<tr>
<th></th>
<th>0 texts</th>
<th>1-5 texts</th>
<th>6-10 texts</th>
<th>11-20 texts</th>
<th>over 20 texts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Texts</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Calls</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

I am going to use pie charts because there are a different number of people in each group.

![30 and under texts](image1)

![over 30 texts](image2)

These charts show that the over 30s do a lot less texting than the younger ones. About three quarters of the over 30s text either 0 or between 1 and 5 times a day. Of the older ones about three quarters text 6 times or more a day.
The difference between these charts is that there are a lot more under 30s making between 6 and 10 calls a day compared to the over 30s.

**Conclusion**

My results show that the majority of people own a mobile phone but my results may not be accurate. I would have to ask more people and maybe more older people to be more sure. I also found that people text more than they make calls.

But it might be that the people that make lots of texts do not make many calls and those that make lots of calls do not make many texts. If I were to do some more work on this I would collect more accurate data about texts and calls and draw a scatter graph to see if this is true.
Older people tend to use calls more than texts compared with the younger people. This might be because they only use their phones to ring people to talk to them and it is mainly teenagers that text a lot because they want to send messages to their friends. It might also be because they have a lot of free texts and calls cost money and they are a bit short of money. If I were to do more work I could find out if the older people who use texts have teenage children who keep them up to date.

I could look up figures on the internet to see if they agree with my findings.

Commentary on Learner B’s work

In this solution there are still areas that could be improved on, for example in the interpretation of the pie charts, but the work would still be Level 2. It is not a perfect solution for the level. However, the piece of work reaches Level 2 because the learner has tackled it as a single complex coherent problem, made decisions about data collection and analysis that help make the results more valid, and interpreted the results with reference to a real-life context. The choices made were more appropriate than those made in the Level 1 piece of work and there was a detailed conclusion that drew it together and suggested reasons.
6. Resources

Contents
6.1 Paper-based materials
6.2 Websites
6.3 Resources designed for schools: Key Stages 3 and 4
6.4 Professional organisations

Most centres will already have a wealth of materials and learning resources. Several recent initiatives have produced free high quality learning and teaching resources for mathematics. Many of these materials provide contextualised opportunities to develop learners’ mathematical techniques.

Recent materials include:

- *Improving learning in mathematics* (see below)
- *Thinking through mathematics* (page 120)
- *Materials for embedded learning* (Skills for Life) (page 121)
- *Resources for key skills learning* (KSSP) (page 121).

All these are available free, either as downloads or in hard copy.

There are also many excellent websites that provide both downloads of paper-based materials and online interactive opportunities that can be used for practising and reinforcing techniques.

6.1 Paper-based materials

*Improving learning in mathematics*

These materials were developed by the DfES Standards Unit and are still commonly known as ‘the Standards Unit box’. They are now part of the National Teaching and Learning Change Programme managed by QIA. There may be a subject learning coach in your centre who will be able to help with ideas for using the materials.

The box includes ‘Resource files’ that contain a wealth of active learning ideas that have proved very successful in developing mathematical techniques and confidence. You will have already met some of these activities in the pages of this document. There is also a box of multimedia resources.

There are a number of other teaching and learning resources, including for construction, engineering, ICT, society, health and development, and creative and media. Many of these contain useful ideas for setting mathematics in context and to develop the process skills for applying the techniques. The resources for construction, engineering, ICT, society, health and development,
and creative and media have been revised to meet the needs of the Diplomas and were published in April 2008.

You can access these materials at http://teachingandlearning.qia.org.uk

The full set of materials includes the topics:

- business
- construction and the built environment
- creative and media
- customer care
- engineering
- enterprise
- health, safety and well-being
- Foundation Learning
- IT
- land-based
- mathematics
- modern foreign languages
- science
- society, health and development.

**Learning Mathematics in Context**

An additional set of teaching and learning resources, *Learning Mathematics in Context*, was published on the QIA website in April 2008. *Learning Mathematics in Context* is designed to help teachers explore aspects of teaching and learning of mathematics within subject and vocational areas. The resources may be viewed at http://teachingandlearning.qia.org.uk/tlp/xcurricula/lmic/.

**Thinking through mathematics**

*Thinking through mathematics* has been developed by the Maths4Life team as a continuation of the work begun by the DfES Standards Unit with *Improving learning in mathematics* (see above).

The ring-binder has an introduction to the approaches as well as sections on collaborative professional development and teaching and learning, and includes a DVD and a CD-ROM of all the materials.

You can order a copy from www.maths4life.org.uk or telephone: 01283 227597.
Materials for embedded learning

The blue ring-binders are a familiar sight in many organisations. There are no fewer than 26 titles available, developed by the Skills for Life Quality Initiative and providing a rich resource of contextualised materials for developing both literacy and numeracy. They cover levels from Entry 1 to Level 2 and are therefore suitable for the full range of functional mathematics learners.

Materials can be downloaded or ordered from the ‘Embedded Learning Portal’ website: www.dcsf.gov.uk/readwriteplus/embeddedlearning/index.cfm or from

DCSF Publications
PO Box 5050
Sherwood Park
Annesley
Nottingham NG15 0DJ
Telephone: 0845 60 222 60
Fax: 0845 60 333 60
Textphone: 0845 60 555 60
Email: dcsf@prolog.uk.com

Resources for key skills learning

Since 2000 the Key Skills Support Programme (KSSP) has produced a range of publications that can be used to develop the techniques and skills required for functional mathematics.

The Resources for key skills learning have been produced primarily for work-based learning and are integrated into different vocational units. Each pack contains teaching and learning materials, ‘How to’ sheets (for application of number and communication) and assessment tasks. They are at levels 1 and 2 and, although the materials have been written to the key skills standards, they can be used to develop learners’ mathematical techniques for functional skills. The sample assessment tasks can be tailored to provide opportunities for developing the process skills.


These resources are now available from http://excellence.qia.org.uk

6.2 Websites

The following websites have been found useful for developing and practising mathematical techniques, process skills and problem solving.

Bear in mind the following notes.

- Some websites disappear or change their web addresses. The URLs given here were active in May 2008.
• In some centres, the network may be set up to block access to certain websites. If this happens, check with your technical support team.

• The list that follows is in alphabetical order of URL and does not imply any ranking by merit or value.

www.bbc.co.uk/schools/gcsebitesize/maths

**BBC bitesize** is useful for GCSE revision and developing the mathematics techniques required at Levels 1 and 2.

www.bbc.co.uk/keyskills

**BBC key skills** – Material aimed at key skills but equally useful for developing functional mathematics.

www.bbc.co.uk/skillswise

**BBC Skillswise** provides downloadable and online interactive materials for both numeracy and literacy. It was developed to support adult basic skills and is very useful to develop and practise mathematical techniques up to Level 1. It contains a wealth of engaging material for learners of all ages.

www.bowlandmaths.org.uk/

Designed for KS3 mathematics, the Bowland activities are aimed at developing thinking, reasoning and problem solving skills. Many of the initial 23 case study problems are ideal for teaching learners to be functional with mathematics.

www.cimt.plymouth.ac.uk/resources/topical/default.htm

**Centre for innovation in mathematics teaching**

Mainly generic topics but a few that may be adapted to suitable contexts and useful for developing both mathematical and process skills.

www.dcsf.gov.uk/readwriteplus/embeddedlearning

The **Embedded Learning Portal** has a full range of Skills for Life learning materials, many of which can be used for developing mathematics techniques for a wide range of learners and levels. There is a list of materials and details of how to obtain free copies.

www.dcsf.gov.uk/readwriteplus/Learning_Materials_Main

The **Skills for Life materials** provide resources that are paper-based and available on CD-ROM. They support literacy, numeracy and ESOL learners at Entry levels, and levels 1 and 2. They are referenced to the adult literacy and numeracy core curriculum and contain clear links to Skills for Life and key skills. They are available to download from the website.
The Raising Standards Guides produced by the DfES Skills for Life Strategy Unit, are intended to help practitioners and managers improve the quality of teaching and managing Skills for Life provision by using the five Common Inspection Framework questions for their particular context. They have now been made available as interactive versions, and can be accessed at the URL above.

http://excellence.qia.org.uk/

The Excellence Gateway is an online service for post-16 learning and skills providers. There are examples of good practice, networks to support self-improvement, suppliers of improvement services, plus tools and materials to support teaching and learning.

www.keyskills4u.com/

KeySkills4u is an online resource for application of number, communication and ICT with some excellent materials to develop mathematical techniques at levels 1 and 2. There are a range of materials including interactive learning, self-assessment and application material and interactive games that are useful for developing process skills.

www.keyskills4u.com/tutorguides/

Keyskills4u Tutor guide includes examples of lesson plans.

www.maths4life.org

Maths4life is a national project sited in the National Centre for Excellence in Mathematics (see below). The project aims to stimulate a positive approach to teaching and learning in adult numeracy and mathematics. In the resources section, there are links to materials on teaching and learning, research and a range of resources.

www.moneymatterstome.com

Money Matters to Me (developed by NIACE) provides a detailed suite of material linked to personal finance.

www.move-on.org.uk/index.asp

Move-on offers skills development materials and practice tests for AoN, numeracy and communication and literacy suitable for functional mathematics development,
www.ncetm.org.uk

The National Centre for Excellence in the Teaching of Mathematics provides a wide range of support, advice and resources to enhance mathematics teaching. Resource material will be developed over time and added to the existing work there.

www.nln.ac.uk

This is the National Learning Network website. You need to register on this site but you can then access some numeracy and other mathematics material.

www.nrdc.org.uk


www.nrich.maths.org/public/index.php

Subtitled ‘Enriching mathematics’, this website contains a wide range of mathematics problems, games and articles. Each month has a different theme and back issues provide a wealth of material which will repay investigation. Start with the home page, which has links to some interesting activities including the Millennium Mathematics Project resource from Cambridge University.

www.raftarget.com/raf

This is the RAF mathematics mission website. Click on the ‘Learners area’ tab to access the online interactive classroom. The site also gives access to:

- ‘The Mathematics Mission’ CD-ROM
  A CD-ROM containing curriculum-related mathematics activities based around a tour of a virtual RAF base.

- The RAF Mathematics Workshop Tour
  An interactive classroom-based mathematics workshop for lower-ability 14–16 year olds which is available to visit UK secondary schools.

www.s-cool.co.uk/default.asp

S-Cool is a revision site for GCSE and A level.
www.skillworkshop.org/about.htm

The Skills workshop on Maggie Harnew's website from Abingdon and Witney College offers shared resources that are principally designed for developing techniques.

www.totallyskilled.org.uk/awardingBody/ab.htm

Totally Skilled, the Embedding Skills for Life and Key Skills in Vocational Qualifications Project website, contains examples of skills audits and task analyses. Resources developed by awarding bodies for this DfES project include support for centres of identifying literacy, language and numeracy skills in vocational courses.

6.3 Resources designed for schools: Key Stages 3 and 4

The resources produced by the National Strategies (Secondary), which are available from the DfES Standards site, support the development of functional mathematics. The early resources were designed for Key Stage 3 learners, but are still relevant for those in Key Stage 4.

www.standards.dfes.gov.uk/secondary/keystage3/respub/

The titles listed are a selection from what is available. The order in which they appear here does not imply any ranking by merit or value.

Mathematics planning toolkit CD-ROM: Key Stage 4

This is included in the pack 'Mathematics subject leader development materials, summer 2007, ref 00277-2007PCK-EN. It, and the accompanying handbook 'Mathematics at Key Stage 4: developing your scheme of work, ref 00049-2007BKT-EN. Copies can be ordered from Prolog: tel. 0845 60 222 60

Assessing pupils’ progress in mathematics at Key Stage 3

Ref 00007-2007FLR-EN

Copies can be ordered from Prolog: tel. 0845 60 222 60

Progression maps

These are part of a suite of resources to support intervention in secondary schools. They can be accessed at:

www.standards.dfes.gov.uk/secondary/keystage3/all/respub/ws_intvsec
Teaching mental mathematics from level 5 series
Details, downloads and ordering instructions are available at www.standards.dfes.gov.uk/secondary/keystage3/all/respub/ma_tmml5up

Using ICT to address ‘hard to teach’ concepts in English and mathematics (June 2007)
ICT offers the potential to transform teaching and learning. This project explores the use of ICT to support the teaching of identified ‘hard to teach’ concepts in English and mathematics.

The project was started in September 2006 and based on action research in classrooms. The research is presented in a series of case studies linked to relevant resources and descriptions of the journeys taken by the teachers in developing the use of ICT in their classrooms. The work they produced, along with some reflections on the experience of the teachers and their learners, is available from this website.

Mathematics study modules (July 2004)
These are ten mathematics study modules, designed for an individual teacher or group of teachers, which have been produced by the mathematics strand of the Key Stage 3 National Strategy. They are intended for teachers who would like to reinforce, confirm and extend their knowledge of the Key Stage 3 mathematics curriculum and to develop their teaching skills.

Interacting with mathematics in Key Stage 3 – constructing and solving linear equations (May 2004)
This material, designed to be used with the Framework for teaching mathematics: Years 7, 8 and 9, provides guidance on developing progression in the teaching of constructing and solving linear equations. Although specific in this focus, it illustrates an approach that is designed to serve the broader purpose of developing the teaching of all aspects of algebra.

Interacting with mathematics in Key Stage 3 – enhancing proportional reasoning (March 2004)
This series of training and school-based materials aims to support mathematics departments in planning for teaching that engages and challenges learners, developing mathematical reasoning and so raise standards of achievement in Key Stage 3.

Interactive Teaching Programs (ITPs) (February 2004)
You will find some mathematics ITPs, created for Key Stage 3 pupils within the National Numeracy Strategy (NNS) programme, and covering a range of mathematical techniques.
Intervention Strategy Key Stage 3 (May 2003)
Intervention is targeted at pupils who are working below national expectations but who have the potential to meet the expectations for their age group if they are given timely support and motivation.

Mathematics vocabulary flashcards (March 2003)
The flashcards have all the words from the vocabulary checklist in section 5 of the Framework for teaching mathematics: Years 7, 8 and 9. They are provided in Word format to allow for individual editing and printing. The vocabulary words are grouped by years and in six sections:

- Algebra
- Handling data
- Numbers and number systems
- Shape, space and measures
- Applying mathematics and solving problems
- Calculations.

Interacting with mathematics in Year 9 – geometrical reasoning (Nov 2002)
This series of training and school-based materials aims to support mathematics departments in planning for teaching that engages and challenges learners, developing mathematical reasoning and so raise standards of achievement in Year 9.

Interacting with mathematics in Year 9 – proportional reasoning (Nov 2002)
This unit is a sequel to the Year 8 multiplicative relationships unit. It provides an opportunity to revise, consolidate and extend ideas introduced in Year 8 and to make links to other mathematical strands, particularly shape and space. Making such links, especially with visual contexts, can help learners to understand proportion.

Teaching able, gifted and talented pupils (September 2002)
This series of optional modules aims to help schools evaluate and develop provision for able, gifted and talented learners. Schools and departments can choose to use some or all of the modules within a planned programme of professional development.

Securing improvement – the role of subject leaders (May 2002)
This booklet is intended to support subject leaders, and in particular to identify core tasks and areas for development.
Teaching and learning functional mathematics: 6. Resources

www.censusatschool.ntu.ac.uk
This is a huge site inviting all kinds of statistical work with real data.

www.1000problems.org
This is an interesting site with problems related to mathematics across the curriculum.

6.4 Professional organisations

The Association of Teachers of Mathematics
www.atm.org.uk

The Association for Achievement and Improvement through Assessment
www.aaia.org.uk

The Mathematical Association
www.m-a.org.uk

NANAMIC (National Association for Numeracy and Mathematics in Colleges)
www.nanamic.org.uk
Teaching and learning functional mathematics: 6. Resources

Acronyms

BSA
Basic Skills Agency. The Basic Skills Agency has merged with the National Institute of Adult and Continuing Education (NIACE) and will work in alliance with Tribal. Its full name is now 'The Basic Skills Agency at NIACE'. See www.niace.org.uk

CBI
Confederation of British Industry. A not-for-profit organisation, incorporated by Royal Charter in 1965. It represents the business sector in the UK, provides membership services, conducts research and provides 'a voice for business' at national level. See www.cbi.org.uk

CEL
Centre for Excellence in Leadership. CEL’s remit has been to foster and support leadership improvement, reform and transformation throughout the sector. On 1 October 2008, CEL and QIA (the Quality Improvement Agency) transferred their operations to LSIS (the Learning and Skills Improvement Service). See www.centreforexcellence.org.uk

CPD
Continuing professional development.

DCSF
Department for Children, Schools and Families. Established in June 2007; successor organisation to DfES. Responsible for functional skills policy. See www.dcsf.gov.uk

DDP
Diploma Development Partnerships. There are 17 Diploma Development Partnerships – one for each Line of Learning – developing content for each of the Diplomas. See www.qca.org.uk/qca_13915.aspx

DfES
Department for Education and Skills. In June 2007, divided into DCSF and DIUS.

DIUS
Department for Innovation, Universities and Skills. Established in June 2007; successor organisation to DfES. Responsibility for key skills and Skills for Life policy. See www.dius.gov.uk
Entry 1, Entry 2, Entry 3
Entry levels in the adult literacy, adult numeracy, adult ICT and ESOL core curricula.

**EFL**
English as a Foreign Language.

**ESOL**
English for Speakers of Other Languages.

**FLT**
Foundation Learning Tier. The umbrella term for all provision below Level 2 that is taken by learners over the age of 14. It therefore encompasses what is currently categorised as pre-Entry, Entry level (split into Entry levels 1, 2 and 3) and Level 1.

**ILP**
Individual Learning Plan. Document used to plan and record a student’s learning.

**Jobcentre Plus**
Government agency that provides help and advice on jobs and training for people who can work and financial help for those who cannot; helps employers to fill vacancies. Part of the Department of Work and Pensions (DWP). See [www.jobcentreplus.gov.uk/](http://www.jobcentreplus.gov.uk/)

**Key Stage 3**
Usually, the first three years of secondary education – Years 7, 8 and 9 – but sometimes condensed.

**Key Stage 4**
Years 10 and 11 of secondary education.

**KSSP**
Key Skills Support Programme. The KSSP at LSN, in partnership with Learning for Work and CfBT Education Trust, supported the delivery and implementation of key skills in schools, colleges, work-based learning and adult learning until 31 March 2008. Resources produced by KSSP can still be found at [www.keyskillssupport.net](http://www.keyskillssupport.net)

**LA**
Local Authority, the education function of which is now incorporated into ‘Integrated Children’s Services’.
learndirect
The largest provider of e-learning in the world. Aims to enable adults without a Level 2 or Skills for Life qualification to gain the skills and qualifications they need to find a job or to achieve and progress at work. See www.learndirect.co.uk

LLN
Literacy, Language, Numeracy.

LLUK
Lifelong Learning UK. Responsible for the professional development of all those working in libraries, archives and information services, work-based learning, higher education, further education and community learning and development. See www.lluk.org.uk

LLU+
National consultancy and professional development centre for staff working in the areas of literacy, numeracy, dyslexia, family learning and ESOL. See www.lsbu.ac.uk/lluplus

LSC
Learning and Skills Council. Responsible for funding and planning education and training for learners over 16 years old in England. See www.lsc.gov.uk

LSIS
Learning and Skills Improvement Service. The Quality Improvement Agency (QIA) and the Centre for Excellence in Leadership (CEL) have come together to form LSIS, the new sector-led organisation dedicated to supporting excellence and leadership development in the further education and skills sector. From 1 October 2008, LSIS will provide programmes hitherto managed by CEL and QIA.

LSN
Learning and Skills Network. Independent not-for-profit organisation delivering quality improvement and staff development programmes that support specific government initiatives, through research, training and consultancy; and by supplying services directly to schools, colleges and training organisations. See www.lsneducation.org.uk

NAA
National Assessment Agency. The Agency supports the secure delivery of the public examinations system (eg GCSEs and A levels) and develops and delivers national curriculum assessments. See www.naa.org.uk
NCSL
The National College for School Leadership exists to help to make a difference to the lives and the life chances of children and young people through the development of world-class school leaders. See www.ncsl.org.uk

NIACE
National Institute of Adult Continuing Education – England and Wales. Non-governmental organisation working for more and different adult learners. See www.niace.org.uk

NSS
The National Strategies (Secondary). Formerly Secondary National Strategies (SNS). NSS is part of the Government’s major reform programme for transforming secondary education to enable children and young people to attend and enjoy school, achieve personal and social development and raise educational standards in line with the ‘Every Child Matters’ agenda. See www.standards.dfes.gov.uk/secondary/about/

NVQ
National Vocational Qualification. NVQs are work-related, competence-based qualifications, accredited by QCA and included in the National Qualifications Framework. See www.qca.org.uk/14-19/qualifications/index_nvqs.htm

OECD
Organisation for Economic Cooperation and Development. The OECD groups thirty member countries sharing a commitment to democratic government and the market economy. See www.oecd.org

Ofqual
Ofqual is the new regulator of qualifications, tests and examinations in England. Ofqual will regulate qualifications in England – work which was previously done by the Qualifications and Curriculum Authority (QCA). See www.ofqual.org.uk

Ofsted
Non-ministerial government department responsible for inspecting and regulating the care of children and young people, and education and skills for learners of all ages. See www.ofsted.gov.uk

QCA
Qualifications and Curriculum Authority. Non-departmental public body, sponsored by government. With the introduction of Ofqual, QCA will be known as the Qualifications and Curriculum Development Agency (QCDA) and will become an agency for developing curriculum, assessment and qualifications. See www.qca.org.uk
QCDA
See QCA.

QIA
Quality Improvement Agency. Non-departmental public body, working across the entire learning and skills sector. On 1 October 2008, QIA and CEL (the Centre for Excellence in Leadership) transferred their operations to LSIS (the Learning and Skills Improvement Service). See www.qia.org.uk

QTLS
Qualified Teacher Learning and Skills. Non-subject-specific qualifications that give qualified teacher status; effective from September 2007.

QTS
Qualified Teacher Status – awarded to a teacher who is fully qualified in terms of training, certification and experience.

Skills for Life
National strategy for improving adult literacy and numeracy skills in England. See www.dfes.gov.uk/readwriteplus and www.sflip.org.uk

SSAT
The Specialist Schools and Academies Trust is the leading national body for secondary education in England, and delivers the Government’s Specialist Schools and Academies programme. Is responsible for CPD for the Diplomas. See www.specialistschools.org.uk

SSC
Sector Skills Council. SSCs are independent, employer-led UK-wide organisations licensed by the Secretary of State for Innovation, Universities and Skills to tackle the skills and productivity needs of their sector throughout the UK. See www.sscalliance.org

TDA
Training and Development Agency for Schools: responsible for funding the provision of teacher training in England, and providing information and advice on teaching as a career. See www.tda.gov.uk

Ufi
University for industry – the organisation behind learndirect. It has a mission to use technology to transform the skills and employability of the working population, in order to improve the UK’s productivity. See www.ufi.com